***Adding and Subtracting Rational Numbers***

The rules for adding and subtracting integers and fractions also apply to adding and subtracting rational numbers.

***Rules for adding and subtracting rational numbers***

* When adding numbers with the same signs, add the absolute value of each number and take the common sign.
* When adding numbers with different signs, subtract the smaller absolute value from the larger absolute value and take the sign of the larger absolute value.
* To subtract, change to adding the opposite and follow the rules for adding signed numbers.
* To add or subtract fractions, you must have a common denominator.
* To add or subtract decimals, it is very important that you are adding values together for the same place value, which means you, must line up the decimal when adding numbers with decimals.

 **Sample Problem 1**: **Find each sum or difference.**

|  |  |  |  |
| --- | --- | --- | --- |
| **a.** | $$ \left(-\frac{4}{5}\right)+\frac{1}{2}=$$ |  **b.** | $$ 4\frac{2}{3}-\left(-3\frac{1}{6}\right)=$$ |
|  | $$ \left(-\frac{4}{5}\right)+\frac{1}{2}=$$$$=\left(-\frac{8}{10}\right)+\frac{5}{10}=$$$$=-\frac{3}{10}$$ |  | $$ 4\frac{2}{3}-\left(-3\frac{1}{6}\right)=$$$$=4\frac{2}{3}+3\frac{1}{6}=$$$$=4\frac{4}{6}+3\frac{1}{6}=$$$$=7\frac{5}{6}$$ |
| **c.** | $$ 2.56+\left(-3.22\right)-4.2=$$ |  **d.** | $$ 6.78-\left(-5.21\right)-2.1=$$ |
|  | $$ 2.56+\left(-3.22\right)-4.2=$$$$=2.56-3.22-4.2=$$$$=-1.34-4.2=$$$$=-5.54$$ |  | $$ 6.78-\left(-5.21\right)-2.1=$$$$=6.78+5.21-2.1=$$$$=11.99-2.1=$$$$=9.89$$ |

***Multiplication and Division of Rational Numbers***

***Rules for multiplying rational numbers***

If the numbers have the same signs then the product will be positive.

$$\left(+\right)\*\left(+\right)=(+)$$

$\left(-\right)\*\left(-\right)=(+)$

If the numbers have different signs then the product will be negative**.**

$$\left(-\right)\*\left(+\right)=\left(-\right)$$

$$\left(+\right)\*\left(-\right)=\left(-\right)$$

***Multiplicative Properties:***

|  |  |  |
| --- | --- | --- |
| **Multiplicative Identity Property**  | The product of a number and 1 is the number.  | $$n\*1=n$$ |
| **Multiplicative Property of** $0$ | The product of a number and zero is zero.  | $$n\*0=0$$ |
| **Multiplicative Inverse Property**  | The product of a number and its reciprocal is one. | $$n\*\frac{1}{n}=1$$ |
| **Multiplicative Property of** $-1$ | The product of a number and negative one is the opposite of the number. | $$n\*\left(-1\right)=-n$$ |

**Sample Problem 2**: **Find each product.**

|  |  |  |  |
| --- | --- | --- | --- |
| **a.**  | $$ \frac{3}{11}\*\left(\frac{-5}{6}\right)=$$ | **b.**  | $$ 3\frac{3}{10}\*\left(-15\right)=$$ |
|  | $$ \frac{3}{11}\*\left(\frac{-5}{6}\right)=$$$$= \frac{3\*(-5)}{11\*6}=$$$$= \frac{3\*(-5)}{11\*3\*2}=$$$$=\frac{-5}{22}$$ |  | $$ 3\frac{3}{10}\*\left(-15\right)=$$$$= \frac{33}{10}\*\frac{\left(-15\right)}{1}=$$$$= \frac{33\*\left(-15\right)}{10\*1}=$$$$= \frac{33\*5\*\left(-3\right)}{2\*5\*1}=$$$$= \frac{33\*\left(-3\right)}{2\*1}=$$$$= \frac{(-99)}{2}=$$$$=-49\frac{1}{2} $$ |
| **c.**  | $$ 2.34\*\left(-0.21\right)=$$ | **d.**  | $$ \left(-0.04\right)\*\left(-1.6\right)=$$ |
|  | $$ 2.34\*\left(-0.21\right)=$$$$=\frac{234}{100}\*\frac{\left(-21\right)}{100}=$$$$=\frac{234\*\left(-21\right)}{100\*100}=$$$$=\frac{-4,914}{10,000}=$$$$=-0.4914$$ |  | $$ \left(-0.04\right)\*\left(-1.6\right)=$$$$=\frac{-4}{100}\*\frac{-16}{10}=$$$$=\frac{\left(-4\right)\*\left(-16\right)}{100\*10}=$$$$=\frac{64}{1,000}=$$$$=0.064$$ |

***Rules for dividing rational numbers***

If the numbers have the same signs, then the quotient will be **positive.**

$$\left(+\right)÷\left(+\right)=\left(+\right) or \frac{\left(+\right)}{\left(+\right)}= \left(+\right)$$

$$\left(-\right)÷\left(-\right)=\left(+\right) or \frac{\left(-\right)}{\left(-\right)}= \left(+\right)$$

If the numbers have different signs, then the quotient will be **negative.**

$$\left(-\right)÷\left(+\right)= \left(-\right) or \frac{\left(-\right)}{\left(+\right)}=\left(-\right)$$

$$\left(+\right)÷\left(-\right)= \left(-\right) or \frac{\left(+\right)}{\left(-\right)}=\left(-\right)$$

Division is multiplying by the reciprocal.

You only have to use the division rule if at least one of the numbers is a fraction or if the quotient will not be an integer.

**Sample problem 3: Find each quotient.**

|  |  |  |  |
| --- | --- | --- | --- |
| **a.**  | $$ \frac{-2}{15}÷\left(\frac{-2}{3}\right)=$$ | **b.**  | $$ 1\frac{4}{11}÷\left(-5\right)=$$ |
|  | $$ \frac{-2}{15}÷\left(\frac{-2}{3}\right)=$$$$= \frac{-2}{15}\*\left(\frac{3}{-2}\right)=$$$$= \frac{\left(-2\right)\*3}{5\*3\*\left(-2\right)}=$$$$= \frac{1}{5}$$ |  | $$ 1\frac{4}{11}÷\left(-5\right)=$$$$= \frac{15}{11}\*\frac{1}{\left(-5\right)}=$$$$= \frac{15\*1}{11\*\left(-5\right)}=$$$$= \frac{3\*5\*1}{11\*\left(-5\right)}$$$$= \frac{3}{-11}$$$$ $$ |
| **c.**  | $$ -0.4÷\left(-5.2\right)=$$ | **d.**  | $$ \left(-0.36\right)÷12=$$ |
|  | $$ -0.4÷\left(-5.2\right)=$$$$=\frac{-4}{10}÷\frac{\left(-52\right)}{10}=$$$$=\frac{-4}{10}\*\frac{10}{\left(-52\right)}=$$$$=\frac{-4\*10}{10\*\left(-52\right)}=$$$$=\frac{-4}{-52}=$$$$=\frac{\left(-4\right)}{\left(-4\right)\*13}$$$$=\frac{1}{13}$$ |  | $$ \left(-0.36\right)÷12=$$$$=\frac{-36}{100}÷\frac{12}{1}=$$$$=\frac{-36}{100}\*\frac{1}{12}=$$$$=\frac{\left(-3\right)\*12\*1}{100\*12}=$$$$=\frac{-3}{100}=$$$$=-0.03$$ |