The graph of **a continuous function** has no breaks, holes, or gaps. You can trace the graph of a continuous function without lifting your pencil.

One condition for a function to be continuous at is that the function must approach a unique function value as -values approach from the left and right sides. The concept of approaching a value without necessarily ever reaching it is called **a limit.**

If the value of approaches a unique valueas approaches from each side, then the limit of as approaches is

Functions that are not continuous are discontinuous. Graphs that are discontinuous can exhibit:

* Infinite discontinuity (A function has an infinite discontinuity at if the function value increases or decreases indefinitely as approaches from the left and right)
* Jump discontinuity,( A function has a jump discontinuity at if the limits of the function as approaches from the left and right exist but have two distinct values.
* Removable discontinuity, also called point discontinuity (function has a removable discontinuity if the function is continuous everywhere except for a hole at .

**Continuity Test**

A function is continuousat if it satisfies the following conditions**.**

1. is defined at **.** exists.
2. approaches the same function value to the left and right of
3. The function value thatapproaches from each side ofis

**Sample Problem 1: Determine whether each function is continuous at the given -values. Justify using the continuity test. If discontinuous, identify the type of discontinuity.**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **a.** |  | |  |  |  |  | | --- | --- | --- | --- | |  |  |  |  | |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | |  |  |  |  | |  |  |  |  | |
| **b.** |  | |  |  |  |  | | --- | --- | --- | --- | |  |  |  |  | |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | |  |  |  |  | |  |  |  |  | |
| **c.** |  | |  |  |  |  | | --- | --- | --- | --- | |  |  |  |  | |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | |  |  |  |  | |  |  |  |  | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **d.** |  | |  |  |  |  | | --- | --- | --- | --- | |  |  |  |  | |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | |  |  |  |  | |  |  |  |  | |

**Intermediate Value Theorem**

If is a continuous function and and there is a value such that is between and then there is a number, such that and

**The Location Principle**

If is a continuous function and and have opposite signs, then there exists at least one value , such that and . That is, there is a zero between and

**Sample Problem 2**: **Determine between which consecutive integers the real zeros of function are located on the given interval.**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **a.** |  | |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |   is positive and is negative**,**    is negative and is positive    has zeros in intervals: |

**End Behavior**

The end behavior of a function describes what the -values do as becomes greater and greater.

When becomes greater and greater, we say that approaches infinity, and we write.

When becomes more and more negative, we say that approaches negative infinity, and we write.

The same notation can also be used with or and with real numbers instead of infinity.

Left - End Behavior (as becomes more and more negative):

Right - End Behavior (as becomes more and more positive):

The values may approach negative infinity, positive infinity, or a specific value.

**Sample Problem 3**: **Use the graph of each function to describe its end behavior. Support the conjecture numerically.**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **a.** |  | From the graph, it appears that:  asand– as    The table supports this conjecture.   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  |  |  |  |  |  | |  |  |  |  | **-** | **-** | |
| **b.** |  | From the graph, it appears that:  as and as    The table supports this conjecture.   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  |  |  |  |  |  | |  |  |  |  |  |  | |

**Increasing, Decreasing, and Constant Functions**

A function is increasing on an interval if and only if for every and contained in , , whenever .

A function is decreasing on an interval if and only if for every and contained in , whenever .

A function remains constant on an interval if and only if for every and contained in , whenever .

Points in the domain of a function where the function changes from increasing to decreasing or from decreasing to increasing are called **critical points.**

**Sample Problem 4**: **Determine the interval(s) on which the function is increasing and the interval(s) on which the function is decreasing.**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **a.** |  | From the graph, it appears that:  A function is decreasing for  A function is increasing for    The table supports this conjecture.   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | |  |  |  |  |  |  |  | |  |  |  |  |  |  |  | |
| **b.** |  | From the graph, it appears that:  A function is increasing:  A function is decreasing:  The table supports this conjecture.   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | |  |  |  |  |  |  |  | |  |  |  |  |  |  |  | |