**Order of Operations**



Numerical expressions are evaluated correctly by following a specific order of operations. Some expressions don’t involve **grouping symbols**, such as parentheses and the like, but some have these. For numerical expressions with no grouping symbols, we make use of the **MDAS** rule. What will happen if numerical expressions have these grouping symbols? Will the rule change?

**Grouping Symbols**

Reading and analyzing a verbal statement is very important in writing a correct numerical expression. There are instances where some numbers and operations are **grouped together** using symbols.

**Parentheses** are not the only grouping symbols used, there are other symbols that you need to know.

**What are these symbols?**

**Braces**

{ }

[ ]

( )

**Brackets**

**Parentheses**

**The Order of Symbols**

Aside from operations, symbols such as parentheses, brackets, and braces, follow a specific order too!

**First, you carry out the operation inside the PARENTHESES.**

**Then, you carry out the operation inside the BRACKETS.**

**Lastly, you carry out the operation inside the BRACES.**

**Evaluating Numerical Expressions with Parentheses**

The word **EVALUATE** means to “**calculate**” or to “**get**” the value of a given expression. This time, the numbers and operations in a numerical expression are grouped using the different grouping symbols. How are these types of numerical expressions evaluated?

**Example:**

Paul and Ana were asked to evaluate the numerical expression:

$$4×\left(2+5\right)-9÷3$$

Their solutions are shown below:

**Ana**

**Paul**

$$4×\left(2+5\right)-9÷3$$

$$8+5-9÷3$$

$$13-9÷3$$

$$13-3$$

$$10$$

$$4×\left(2+5\right)-9÷3$$

$$4×7-9÷3$$

$$28-9÷3$$

$$28-3$$

$$25$$

There could only be **ONE** correct answer. Who do you think did it right? Is it Paul or Ana?

To ensure that the answer is “**valid**” and “**accurate**,” there are certain steps to follow. For numerical expressions without grouping symbols, we use the **MDAS** rule. The PEMDAS rule is used for numerical expressions involving grouping symbols such as **parentheses**, **brackets**, and **braces**.

**What is PEMDAS rule???**

**PEMDAS Rule**

Applying the **PEMDAS** rule is very important in evaluating numerical expressions. Why is PEMDAS very important? What does it stand for?

s

A

D

M

E

P

**Multiplication**

**Exponents**

**Parentheses**

**Subtraction**

**Addition**

**Division**

Following the **PEMDAS** rule is very important especially if your goal is to get the correct answer. Below are the steps to make sure that you are doing it right!!!

**Step #1**

**You work on the numbers and operations inside the grouping symbols first. Work on the calculation inside the parentheses, the calculation inside the brackets, and the calculations inside the braces.**

**Step #2**

**Find the value of numbers with exponents (if there are any).**

**Step #3**

**If all the calculations inside the grouping symbols are done, you may now start working on the calculations from left to right like the arrows below.**

**Step #4**

**Work with MULTIPLICATION or DIVISION, whichever comes first, from LEFT to RIGHT.**

**Step #5**

**Work with ADDITION or SUBTRACTION, whichever comes first, from LEFT to RIGHT.**

So… going back to Paul and Ana’s solution:

**Ana**

**Paul**

$$4×\left(2+5\right)-9÷3$$

$$8+5-9÷3$$

$$13-9÷3$$

$$13-3$$

$$10$$

$$4×\left(2+5\right)-9÷3$$

$$4×7-9÷3$$

$$28-9÷3$$

$$28-3$$

$$25$$

Who did it right? Who followed the steps properly?

****

 **Paul did it right!!!**

Let’s find out how he did it right!

**Follow the MDAS rule working from LEFT to RIGHT.**

**Multiply or Divide**

**(whichever comes first)**

**Add or Subtract**

**(whichever comes first)**

**Work on the calculations inside the parentheses.**

**Paul**

$$4×\left(2+5\right)-9÷3$$

$$4×7-9÷3$$

$$28-9÷3$$

$$28-3$$

$$25$$

On the other hand, ANA failed to follow the PEMDAS rule.

**Not working on the calculations inside grouping symbols first, like parentheses, is totally a WRONG move! Thus, the answer will be INCORRECT ☹**

$$4×\left(2+5\right)-9÷3$$

$$8+5-9÷3$$

$$13-9÷3$$

$$13-3$$

$$10$$

**Sample Problem 1:**

Two students were asked to evaluate the numerical expression below. Who do you think has the correct answer? Why do you think so?

$$\left(8+7\right)÷3+(4-2)×3$$

**Student B**

**Student A**

$$\left(8+7\right)÷3+(4-2)×3$$

$$15÷3+(4-2)×3$$

$$15÷7-2×3$$

$$15÷5×3$$

$$15÷15$$

$$1$$

$$\left(8+7\right)÷3+(4-2)×3$$

$$15÷3+(4-2)×3$$

$$15÷3+2×3$$

$$5+2×3$$

$$5+6$$

$$11$$

**Sample Problem 2:**

Evaluate the numerical expression $\left(10+2\right)÷4×[\left(15÷3\right)-4]$ using the PEMDAS rule.

**The FUNNEL Method**

If the steps involved in evaluating numerical expressions confuse you. Like when you see the numbers and they all dance in your head. There is a method that you can use that will help you get the correct answer one step at a time  ☺!

**The Funnel Method**

1. **Write the expression horizontally.**
2. **Determine the operation that should be done first, following the PEMDAS rule, and underline it.**
3. **Perform the said operation and rewrite the expression the way it appeared in the original expression.**
4. **Underline the next operation following the MDAS rule.**
5. **Continue performing the operations one at a time, rewrite the expression after each step until you’ve completed all the operations… and you have one value left.**

**Example:**

Evaluate the numerical expression $(3×4)-\left[\left(24÷2\right)-7\right]+6$ using the **Funnel Method**.

**Solution:**

$$(3×4)-\left[\left(24÷2\right)-7\right]+6$$

$$(3×4)-\left[12-7\right]+6$$

$$(3×4)-5+6$$

$$12-5+6$$

$$7+6$$

$$13$$

**Sample Problem 3:**

Evaluate the numerical expression $[10-4÷\left(9-5\right)]-3×3$using the funnel method.

**Order of Operations in the Real World**

The **PEMDAS** rule is also used to solve real-life problems. These problems happen daily without us realizing we are using such a rule. Below is an example:

**Example:**

A shoe store is on sale, and it gives a $30 discount for each pair of shoes. Mel and his friend bought 4 pairs of shoes with an original price of $80, and they decided to share the cost.

1. Write a numerical expression to represent the above.

 Solution:

$$(80-30)×4÷2$$

1. How much should each of them pay?

 Solution:

$$(80-30)×4÷2$$

$$50×4÷2$$

$$200÷2$$

$$100$$

They have to pay $100 each.

**Sample Problem 4:**

Kyle saved for a year, and he was able to raise $2000. He gave his mom $500 and divided the remaining amount into 4 parts and kept 1 part for himself. Then, he bought a new shirt that cost $100.

1. Write a numerical expression to represent the above.
2. How much money did he have left?