**Order of Operations**



Numerical expressions are evaluated correctly by following a specific order of operations. Some expressions don’t involve **grouping symbols**, such as parentheses and the like, but some have these. For numerical expressions with no grouping symbols, we make use of the **MDAS** rule. What will happen if numerical expressions have these grouping symbols? Will the rule change?

**Grouping Symbols**

Reading and analyzing a verbal statement is very important in writing a correct numerical expression. There are instances where some numbers and operations are **grouped together** using symbols.

**Parentheses** are not the only grouping symbols used, there are other symbols that you need to know.

**What are these symbols?**

**Braces**

{ }

[ ]

( )

**Brackets**

**Parentheses**

**The Order of Symbols**

Aside from operations, symbols such as parentheses, brackets, and braces, follow a specific order too!

**First, you carry out the operation inside the PARENTHESES.**

**Then, you carry out the operation inside the BRACKETS.**

**Lastly, you carry out the operation inside the BRACES.**

**Evaluating Numerical Expressions with Parentheses**

The word **EVALUATE** means to “**calculate**” or to “**get**” the value of a given expression. This time, the numbers and operations in a numerical expression are grouped using the different grouping symbols. How are these types of numerical expressions evaluated?

**Example:**

Paul and Ana were asked to evaluate the numerical expression:

$$4×\left(2+5\right)-9÷3$$

Their solutions are shown below:

**Ana**

**Paul**

$$4×\left(2+5\right)-9÷3$$

$$8+5-9÷3$$

$$13-9÷3$$

$$13-3$$

$$10$$

$$4×\left(2+5\right)-9÷3$$

$$4×7-9÷3$$

$$28-9÷3$$

$$28-3$$

$$25$$

To ensure that the answer is “**valid**” and “**accurate**,” there are certain steps to follow. For numerical expressions without grouping symbols, we use the **MDAS** rule. The PEMDAS rule is used for numerical expressions involving grouping symbols such as **parentheses**, **brackets**, and **braces**.

**What is PEMDAS rule???**

**PEMDAS Rule**

Applying the **PEMDAS** rule is very important in evaluating numerical expressions. Why is PEMDAS very important? What does it stand for?

s

A

D

M

E

P

**Multiplication**

**Exponents**

**Parentheses**

**Subtraction**

**Addition**

**Division**

Following the **PEMDAS** rule is very important especially if your goal is to get the correct answer. Below are the steps to make sure that you are doing it right!!!

**Step #1**

**You work on the numbers and operations inside the grouping symbols first. Work on the calculation inside the parentheses, then the calculation inside the brackets and lastly, the calculations inside the braces.**

**Step #2**

**Find the value of numbers with exponents (if there are any).**

**Step #3**

**If all the calculations inside the grouping symbols are done, you may now start working on the calculations from left to right like the arrows below.**

**Step #4**

**Work with MULTIPLICATION or DIVISION, whichever comes first, from LEFT to RIGHT.**

**Step #5**

**Work with ADDITION or SUBTRACTION, whichever comes first, from LEFT to RIGHT.**

So… going back to Paul and Ana’s solution:

**Ana**

**Paul**

$$4×\left(2+5\right)-9÷3$$

$$8+5-9÷3$$

$$13-9÷3$$

$$13-3$$

$$10$$

$$4×\left(2+5\right)-9÷3$$

$$4×7-9÷3$$

$$28-9÷3$$

$$28-3$$

$$25$$

Who did it right? Who followed the steps properly?

****

 **Paul did it right!!!**

Let’s find out how he did it right!

**Follow the MDAS rule working from LEFT to RIGHT.**

**Multiply or Divide**

**(whichever comes first)**

**Add or Subtract**

**(whichever comes first)**

**Work on the calculations inside the parentheses.**

**Paul**

$$4×\left(2+5\right)-9÷3$$

$$4×7-9÷3$$

$$28-9÷3$$

$$28-3$$

$$25$$

On the other hand, ANA failed to follow the PEMDAS rule.

**Not working on the calculations inside grouping symbols first, like parentheses, is totally a WRONG move! Thus, the answer will be INCORRECT ☹**

$$4×\left(2+5\right)-9÷3$$

$$8+5-9÷3$$

$$13-9÷3$$

$$13-3$$

$$10$$

**Sample Problem 1:**

Two students were asked to evaluate the numerical expression below. Who has the correct answer? Why do you think so?

$$\left(8+7\right)÷3+(4-2)×3$$

**Student B**

**Student A**

$$\left(8+7\right)÷3+(4-2)×3$$

$$15÷3+(4-2)×3$$

$$15÷7-2×3$$

$$15÷5×3$$

$$15÷15$$

$$1$$

$$\left(8+7\right)÷3+(4-2)×3$$

$$15÷3+(4-2)×3$$

$$15÷3+2×3$$

$$5+2×3$$

$$5+6$$

$$11$$

**Solution:** (Answers may vary)

Student B’s solution is correct. He/she followed the PEMDAS rule correctly.

**Sample Problem 2:**

Evaluate the numerical expression $\left(10+2\right)÷4×[\left(15÷3\right)-4]$ using the PEMDAS rule.

Solution:

$$\left(10+2\right)÷4×[\left(15÷3\right)-4]$$

$$12÷4×[\left(15÷3\right)-4]$$

$$12÷4×[5-4]$$

$$12÷4×1$$

$$3×1$$

**3**

**The FUNNEL Method**

 Suppose the steps involved in evaluating numerical expressions confuse you. Like when you see the numbers and they all dance in your head. There is a method that you can use that will help you get the correct answer one step at a time ☺!

**The Funnel Method**

1. **Write the expression horizontally.**
2. **Determine the operation that should be done first, following the PEMDAS rule, and underline it.**
3. **Perform the said operation and rewrite the expression the way it appeared in the original expression.**
4. **Underline the next operation following the MDAS rule.**
5. **Continue performing the operations one at a time, rewrite the expression after each step until you’ve completed all the operations… and you have one value left.**

**Example:**

Evaluate the numerical expression $(3×4)-\left[\left(24÷2\right)-7\right]+6$ using the **Funnel Method**.

**Solution:**

$$(3×4)-\left[\left(24÷2\right)-7\right]+6$$

$$(3×4)-\left[12-7\right]+6$$

$$(3×4)-5+6$$

$$12-5+6$$

$$7+6$$

$$13$$

**Sample Problem 3:**

Evaluate the numerical expression $[10-4÷\left(9-5\right)]-3×3$using the funnel method.

**Solution:**

$$[10-4÷\left(9-5\right)]-3×3$$

$$[10-4÷4]-3×3$$

$$[10-1]-3×3$$

$$9-3×3$$

$$9-9$$

$$0$$

**Order of Operations in the Real World**

1. The **PEMDAS** rule is also used to solve real-life problems. These problems happen daily without us realizing we are using such a rule. Below is an example:

**Sample Problem 4:**

Kyle saved for a year and he was able to raise $2000. He gave his mom $500 and divided the remaining amount into 4 parts and kept 1 part for himself. Then, he bought a new shirt that cost $100.

1. Write a numerical expression to represent the above.
2. How much money did he have left?

Solution:

1. Write a numerical expression to represent the situation.

$$\left(2000-500\right)÷4-100$$

1. How much money did he have left?

$$\left(2000-500\right)÷4-100$$

$$1500÷4-100$$

$$375-100$$

$$275$$

He has $275 left.