**PROPERTIES OF REAL NUMBERS**

Let$ a$, $b$, and $c$ be any real numbers

1. **IDENTITY PROPERTIES**

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|  | **Additive Identity** |  |
|  | The sum of any number and $0$ is equal to the number. Thus, $0$ is called the **additive** **identity**. |
|  | *For any number* $a$*, the sum of* $a$ *and* $0$ *is* $a$*.* | $$a+0=0+a=a$$ |
|  | **Multiplicative Identity** |  |
|  | The product of any number and $1$ is equal to the number. Thus, $1$ is called the **multiplicative identity**. |
|  | *For any number* $a$*, the product of* $a$ *and* $1$ *is* $a$*.* | $$a⋅1=1⋅a=a$$ |

1. **INVERSE PROPERTIES**

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|  | **Additive Inverse** |  |
|  | The sum of any number and its opposite number (its negation) is equal to $0$. Thus, $0$ is called the **additive** **inverse**. |
|  | *For any number* $a$*, the sum of* $a$ *and* $-a$ *is* $0$*.* | $$a+\left(-a\right)=\left(-a\right)+a=0$$ |
|  | **Multiplicative Property of Zero** |  |
|  | *For any number* $a$*, the product of* $a$ *and* $0$ *is* $0$*.* | $$a⋅0=0⋅a=0$$ |
|  | **Multiplicative Inverse** |  |
|  | The product of any number and$ $its reciprocal is equal to $1$. Thus, the number’s reciprocal is called the **multiplicative inverse**. |  |
|  | *For any number* $a$*, the product of* $a$ *and its reciprocal* $\frac{1}{a}$ *is* $1$*.* | $$a⋅\frac{1}{a}=\frac{1}{a}⋅a=1$$ |
|  | *For any numbers*$ \frac{a}{b}$*, where* $b\ne 0$*, the product of* $\frac{a}{b}$*and its reciprocal*$\frac{b}{a}$*is* $1$*.* | $$\frac{a}{b}⋅\frac{b}{a}=\frac{b}{a}⋅\frac{a}{b}=1$$ |

**Sample Problem 1**: Name the property in each equation. Then find the value of $x$.

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|  | $$11⋅x=11$$ | **Multiplicative identity** | $$x=1$$ |
|  | $$x+0=24$$ | **Additive identity** | $$x=24$$ |
|  | $$x⋅4=1$$ | **Multiplicative inverse** | $$x=\frac{1}{4}$$ |
|  | $$x+45=0$$ | **Additive inverse** | $$x=-45$$ |
|  | $$x⋅8=0$$ | **Multiplicative product of zero** | $$x=0$$ |
|  | $$\frac{4}{7}⋅x=1$$ | **Multiplicative inverse** | $$x=\frac{7}{4}$$ |

1. **EQUALITY PROPERTIES**

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|  | **Reflexive**  |  |
|  | Any quantity is equal to itself. |
|  | *For any number* $a$*,* $a=a$*.* | $$a=a$$ |
|  | **Symmetric** |  |
|  | If one quantity equals a second quantity, then the second quantity equals the first quantity. |
|  | *For any numbers* $a$ *and* $b$*, if* $a=b$ *then* $b=a$*.* | $a=b$$b=a$ |
|  | **Transitive** |  |
|  | If one quantity equals a second quantity and the second quantity equals a third quantity, then the first quantity equals the third quantity. |
|  | *For any numbers* $a$*,* $b$*, and* $c$*, if* $a=b$ *and* $b=c$*, then*$ a=c$*.* | $a=b$$b=c$$$a=c$$ |
|  | **Substitution** |  |
|  | A quantity may be substituted for its equal in any expression.  |
|  | *If* $a=b$*, then* $a$ *may be replaced by* $b$ *in any expression.* | $a=b$$3a=3⋅b$ |

**Sample Problem 2**: Evaluate$ x\left(xy-5\right)+y⋅\frac{1}{y}$, if $x=4$ and $y=5$. Name the property of equality used in each step.

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| $$x\left(xy-5\right)+y⋅\frac{1}{y}$$ | $$=$$ | $$4\left(4⋅5-5\right)+5⋅\frac{1}{5}$$ | Substitution: $x=4$ and $y=5$ |
|   | $$=$$ | $$4\left(4⋅5-5\right)+1$$ | Multiplicative inverse: $5⋅\frac{1}{5}=1$ |
|  | $$=$$ | $$4\left(20-5\right)+1$$ | Substitution: $4⋅5=20$ |
|   | $$=$$ | $$4\left(15\right)+1$$ | Substitution: $20-5=15$ |
|   | $$=$$ | $$60+1$$ | Multiplicative identity: $4\left(15\right)=60$ |
| $$x\left(xy-5\right)+y⋅\frac{1}{y}$$ | $$=$$ | $$61$$ | Substitution: $60+1=61$ |

1. **COMMUTATIVE PROPERTIES**

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|  | **Addition**  |  |
|  | The order in which two numbers are added does not change their sum. |
|  | *For any numbers* $a$ *and* $b$*,* $a+b$ *is equal to* $b+a$*.* | $$a+b=b+a$$ |
|  | **Multiplication** |  |
|  | The order in which two numbers are multiplied does not change their product. |
|  | *For any numbers* $a$ *and* $b$*,* $a⋅b$ *is equal to* $b⋅a$*.* | $$ab=ba$$ |

1. **ASSOCIATIVE PROPERTIES**

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|  | **Addition**  |  |
|  | The way three or more numbers are grouped when adding does not change their sum. |
|  | *For any numbers* $a$*,* $b$*, and* $c$*,* $\left(a+b\right)+c$ *is equal to* $a+(b+c)$*.* | $$\left(a+b\right)+c=a+\left(b+c\right)$$ |
|  | **Multiplication** |  |
|  | The way three or more numbers are grouped when multiplying does not change their product. |
|  | *For any numbers* $a$*,* $b$*, and* $c$*,* $\left(a⋅b\right)⋅c$ *is equal to* $a⋅(b⋅c)$*.* | $$\left(a⋅b\right)⋅c=a⋅(b⋅c)$$ |

**Sample Problem 3**: Simplify variable expressions. Show all possible answers.

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|  | $$10+\left(x+4\right)$$ | $$=9+x$$ | $$=x+9$$ |
|  | $$\left(2+x\right)+5$$ | $$=3+x$$ | $$=x+3$$ |
|  | $$3⋅2x$$ | $$=35x$$ |  |
|  | $$\left(x+1\right)+2$$ | $$=x+12$$ | $$=12+x$$ |
|  | $$\left(4\right)\left(3x\right)$$ | $$=18x$$ |  |