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Simplifying Numerical Expressions (Order of Operations)

Guide Notes

SIMPLIFYING NUMERICAL EXPRESSIONS

In Mathematics, to simplify means to make something its simplest form. For example:

$$(-6) + (-5) + 8 \text{ can be simplified as } -3$$

The expression above only involves the **addition** operation. What will happen if a numerical expression involves all four operations, including parentheses, brackets, braces and exponents? To simplify an expression like this, the correct order of operations must be followed. Not doing so can lead into a HUGE disaster.

ORDER OF OPERATIONS

We may encounter four different types of numerical expressions:

- **Expressions involving the four operations**
- **Expressions involving parentheses, brackets, and the four operations**
- **Expressions involving the four operations and exponents**
- **Expressions involving parentheses, brackets, braces, exponents and the four operations**

EXPRESSIONS INVOLVING THE FOUR OPERATIONS

Expressions involving addition, subtraction, multiplication and division, follows the **MDAS** rule.

Multiplication	Division	Addition	Subtraction

- First, do all multiplications and/or divisions, whichever comes first, from left to right.
- Then, do all additions and/or subtractions, whichever comes first, from left to right.

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Example 1: $(-8) + 10 \div (-2) - 5 \times 4$

$$(-8) + 10 \div (-2) - 5 \times 4$$
$$(-8) + (-5) - 5 \times 4$$
$$(-8) + (-5) \quad -20$$
$$-13 - 20$$
$$\mathbf{-33}$$

Example 2: $(-6)(3) \div 9 - 10 - (-12)$

$$(-6)(3) \div 9 - 10 - (-12)$$
$$-18 \div 9 \quad -10 - (-12)$$
$$-2 - 10 \quad - (-12)$$
$$-12 - (-12)$$
$$-12 + 12$$
$$\mathbf{0}$$

Sample Problem 1: Simplify the expressions below.

1. $(-9)(-4) - 6 \times 5 + (-10)$

$$36 - 6 \times 5 + (-10)$$
$$36 - 30 + (-10)$$
$$6 + (-10)$$
$$-4$$

2. $15 - (-8) + 6 \div (-3)(-2)$

$$15 - (-8) + (-2)(-2)$$
$$15 - (-8) + 4$$
$$23 + 4$$
$$27$$

3. $10 + 2 \times (-5) - 16 \div (-4)$

$$10 + (-10) - 16 \div (-4)$$
$$10 + (-10) - (-4)$$
$$0 - (-4)$$
$$4$$

4. $(-25) \div (-5)(5) - 20 + (-5)$

$$(5)(5) - 20 + (-5)$$
$$25 - 20 + (-5)$$
$$5 + (-5)$$
$$0$$

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EXPRESSIONS INVOLVING PARENTHESES, BRACKETS AND THE FOUR OPERATIONS

Expressions involving parentheses, brackets, braces, addition, subtraction, multiplication and division, follows the **PMDAS** rule.

P	M	D	A	S
Parentheses, Brackets and Braces	Multiplication	Division	Addition	Subtraction

- First, remove grouping symbols like **( )**, **[ ]**, **{ }** by working on operations from the innermost part. Start with operations inside the parentheses, followed by the operations inside the brackets and then the operations inside the braces. These symbols disappear when you perform the all the operations (still following the MDAS rule) inside them.

- Then, do all multiplications and/or divisions, whichever comes first, from left to right.

- Lastly, do additions and/or subtractions, whichever comes first, from left to right.

Example 1: $-10[16 \div (-4) + 3] - 4$

$$-10[16 \div (-4) + 3] - 4$$
$$-10[-4 + 3] - 4$$
$$-10[-1] - 4$$
$$10 - 4$$
$$\mathbf{6}$$

Example 2: $[(9 - 3) \div 2(-3)] - [5 + (3)(-2)]$

$$[(9 - 3) \div 2(-3)] - [5 + (3)(-2)]$$
$$[6 \div 2(-3)] - [5 + (-6)]$$
$$[3(-3)] - [-1]$$
$$[-9] - [-1]$$
$$-9 + 1$$
$$\mathbf{-8}$$

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Sample Problem 2: Simplify the expressions below.

1. $9(9 \div 3) + (-4 + 9)$

$$9(3) + 5$$
$$27 + 5$$
$$22$$

2. $24 \div 4(9 - 7) - 6$

$$24 \div 4(2) - 6$$
$$6(2) - 6$$
$$12 - 6$$
$$6$$

3. $2[5 - (-8 \div 4) + 3]$

$$2[5 - (-2) + 3]$$
$$2[7 + 3]$$
$$2[10]$$
$$20$$

4. $[(18 \div 3) + (-5 + 5)2] - 9$

$$[6 + (0)2] - 9$$
$$[6 + 0] - 9$$
$$6 - 9$$
$$-3$$

EXPRESSIONS INVOLVING THE FOUR OPERATIONS AND EXPONENTS

Expressions involving exponents, addition, subtraction, multiplication and division, follows the **EMDAS** rule.

Exponents	Multiplication	Division	Addition	Subtraction

- First, simplify the numbers with exponents. Make sure that all numbers raised to a certain power are calculated.
- Then, do all multiplications and/or divisions, whichever comes first, from left to right.
- Lastly, do all additions and/or subtractions, whichever comes first, from left to right.

Example 1: $5^2 \div 5 + (-5)(5) - 5$

$$5^2 \div 5 + (-5)(5) - 5^2$$
$$25 \div 5 + (-5)(5) - 25$$
$$5 + (-5)(5) - 25$$
$$5 + (-25) - 25$$
$$-20 - 25$$
$$-45$$

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Example 2: $12 - (-2)^4 \div (-16) + 3^2 - (-1)^3$

$$12 - (-2)^4 \div (-16) + 3^2 - (-1)^3$$
$$12 - 16 \div (-16) + 9 - (-1)$$
$$12 - (-1) + 9 - (-1)$$
$$13 + 9 - (-1)$$
$$22 - (-1)$$
$$23$$

Sample Problem 3: Simplify the expressions below.

1. $4^2 - 24 \div (-2)^3 + 8$

$$16 - 24 \div (-8) + 8$$
$$16 - (-3) + 8$$
$$19 + 8$$
$$27$$

2. $(-1)^2(-1)^3 - 1 + (-1)^5$

$$(1)(-1) - 1 + (-1)$$
$$-1 - 1 + (-1)$$
$$-2 + (-1)$$
$$-3$$

3. $(-3)^3 \div 3(-2) - 4^2 \div 2$

$$27 \div 3(-2) - 16 \div 2$$
$$(9)(-2) - 16 \div 2$$
$$-18 - 16 \div 2$$
$$-18 - 8$$
$$-26$$

4. $(5^3)(-1)^5 + 8 \div (-2)^3(2^3)$

$$125(-1) + 8 \div (-8)(8)$$
$$-125 + 8 \div (-8)(8)$$
$$-125 + (-1)(8)$$
$$-125 + (-8)$$
$$-133$$

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EXPRESSIONS INVOLVING PARENTHESES, EXPONENTS AND THE FOUR OPERATIONS

Expressions involving parentheses, brackets, addition, subtraction, multiplication and division, follows the **PMDAS** rule.

Parentheses, Brackets and Braces	Exponents
Multiplication	Division
Addition	Subtraction

- First, remove grouping symbols like **( )**, **[ ]**, **{ }** by working on operations from the innermost part. Start with operations inside the parentheses, followed by the operations inside the brackets and then the operations inside the braces. These symbols disappear when you perform the all the operations (still following the MDAS rule) inside them.

- Next, simplify the numbers with exponents. Make sure that all numbers raised to a certain power are calculated.

- Then, do all multiplications and/or divisions, whichever comes first, from left to right.

- Lastly, do additions and/or subtractions, whichever comes first, from left to right.

Example 1: $(45 + 7) \div (12 \div 3) + 3^2$

$$(45 + 7) \div (12 \div 3) + 3^2$$

$$52 \div 4 + 3^2$$

$$52 \div 4 + 9$$

$$13 + 9$$

$$\mathbf{22}$$

Example 2: $[(75 - 26) \div 7]^2 + [(-2)^2(84 \div 12)]$

$$[(75 - 26) \div 7]^2 + [(-2)^2(84 \div 12)]$$

$$[49 \div 7]^2 + [(-2)^2(7)]$$

$$[7]^2 + [(4)(7)]$$

$$49 + 28$$

$$\mathbf{77}$$

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Sample Problem 4: Simplify the expressions below.

1. $5(5+2^4)-[(5^2\div 5)+(-2)^4-5]$

$$5(5+16)-[(25\div 5)+16-5]$$
$$5(21)-[5+16-5]$$
$$5(21)-16$$
$$105-16$$
$$89$$

2. $-5+3[6\div 2+(2^3\times 5)\div 2^2]-10$

$$-5+3[6\div 2+(8\times 5)\div 4]-10$$
$$-5+3[6\div 2+40\div 4]-10$$
$$-5+3[3+40\div 4]-10$$
$$-5+3[3+10]-10$$
$$-5+3[13]-10$$
$$-5+39-10$$
$$34-10$$
$$24$$