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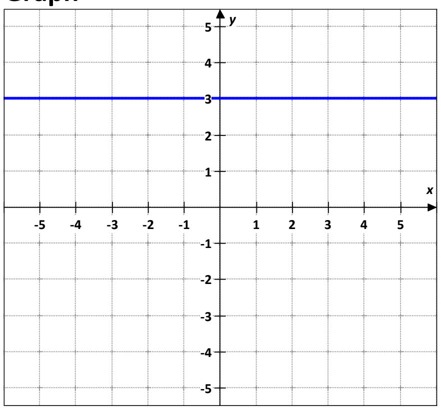
Parent Functions and Transformations Guided Notes

A family of functions is a group of functions with graphs that display one or more similar characteristics.

The Parent Function is the simplest function with the defining characteristics of the family. Functions in the same family are transformations of their parent functions.

Family - Constant Function

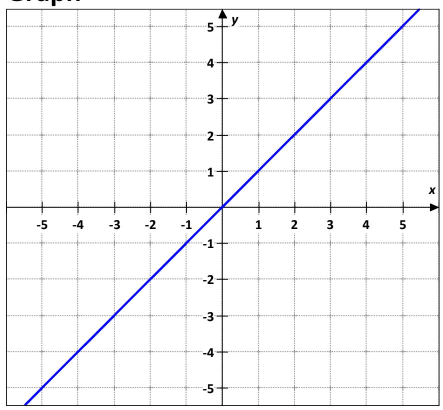
Graph



Rule $f(x) = c$
Domain $= (-\infty, \infty)$
Range $= [c]$

Family - Linear Function

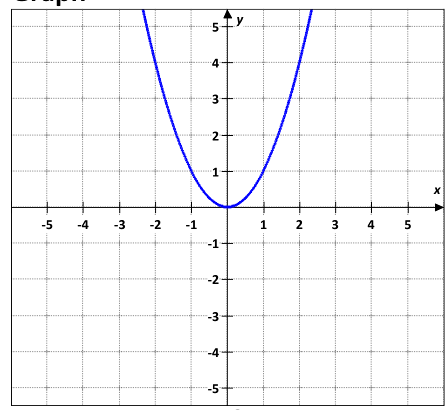
Graph



Rule $f(x) = x$
Domain $= (-\infty, \infty)$
Range $= (-\infty, \infty)$

Family - Quadratic Function

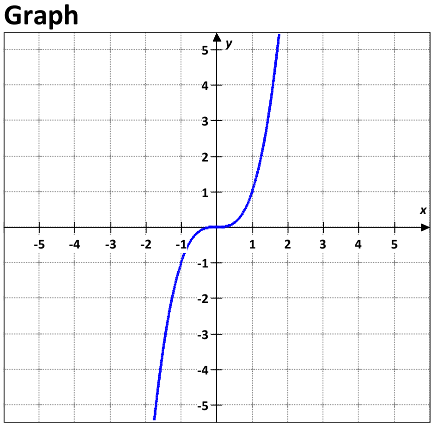
Graph



Rule $f(x) = x^2$
Domain $= (-\infty, \infty)$
Range $= [0, \infty)$

Family - Cubic Function

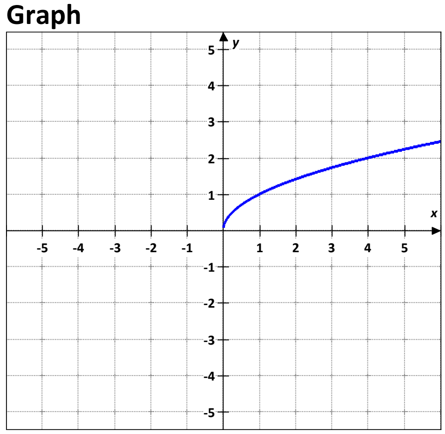
Graph



Rule $f(x) = x^3$
Domain $= (-\infty, \infty)$
Range $= (-\infty, \infty)$

Family - Square Root Function

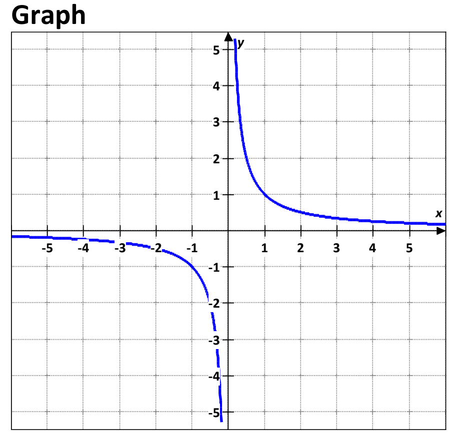
Graph



Rule $f(x) = \sqrt{x}$
Domain $= [0, \infty)$
Range $= [0, \infty)$

Family - Reciprocal Function

Graph



Rule $f(x) = \frac{1}{x}$
Domain $= (-\infty, 0) \cup (0, \infty)$
Range $= (-\infty, 0) \cup (0, \infty)$

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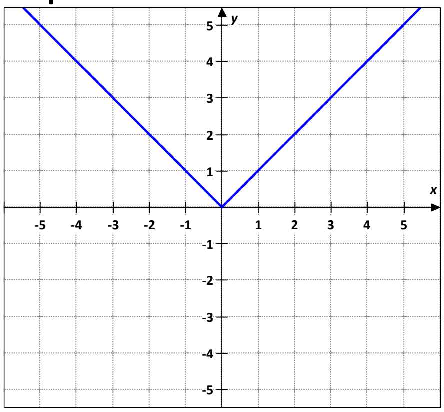
Parent Functions and Transformations Guided Notes

Family – Absolut Value Function

Graph



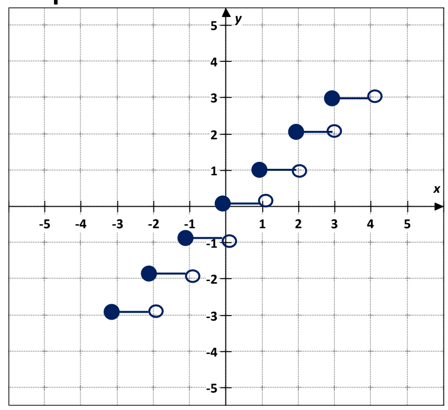
Rule $f(x) = |x| = \begin{cases} -x & if \;\; x < 0 \\ x & if \;\; x \geq 0 \end{cases}$

Domain$= (-\infty, \infty)$

Range $= [\mathbf{0}, \infty)$

Family - Greatest Integer Function

Graph



Rule $f(x) = [\![x]\!]$

Domain$= (-\infty, \infty)$

Range $\boldsymbol{All\ Integer}$

Transformations

Transformations

A change in the size or position of a figure or graph of the function is called a transformation.

Rigid transformations change only the position of the graph, leaving the size and shape unchanged.

	Appearance in Function	Transformation of Graph	Transformation of Point
Vertical Translations	$f(x) \to f(x) + a$ $f(x) \to f(x) - a$	$\boldsymbol{a}$ *units up* $\boldsymbol{a}$ *units down*	$(x, y) \to (x, y + a)$ $(x, y) \to (x, y - a)$
Horizontal Translations	$f(x) \to f(x - b)$ $f(x) \to f(x + b)$	$\boldsymbol{b}$ *units right* $\boldsymbol{b}$ *units left*	$(x, y) \to (x + b, y)$ $(x, y) \to (x - b, y)$
Reflections in x-axes	$f(x) \to -f(x)$	*reflected in the* $\boldsymbol{x}$ ***axis***	$(x, y) \to (x, -y)$
Reflections in y-axes	$f(x) \to f(-x)$	*reflected in the* $\boldsymbol{y}$ ***axis***	$(x, y) \to (-x, y)$

Non rigid transformations distort the shape of the graph.

	Appearance in Function	Transformation of Graph	Transformation of Point
Vertical Dilations	$f(x) \to cf(x) \quad c > 1$ $f(x) \to cf(x) \quad 0 < c < 1$	*expanded vertically* *compressed vertically*	$(x, y) \to (cx, y)$
Horizontal Dilations	$f(x) \to f(dx) \quad d > 1$ $f(x) \to f(dx) \quad 0 < d < 1$	*compressed horizontally* *expanded horizontally*	$(x, y) \to \left(\frac{x}{d}, y\right)$

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Parent Functions and Transformations Guided Notes

Sample Problem 1: Identify the parent function and describe the transformations.

a. $f(x) = (x-1)^2$

Parent : $f(x) = x^2$
Transformation: Translation 1 unit right

b. $f(x) = x^3 - 5$

Parent : $f(x) = x^3$
Transformation: Translation 5 units down

c. $f(x) = -|x+4|$

Parent : $f(x) = |x|$
Transformation: Reflection in x-axis
Translation 4 units left

d. $f(x) = 3x^2 + 7$

Parent : $f(x) = x^2$
Transformation: Expand vertically by a factor of 3
Translation 7 units up

Sample Problem 2: Given the parent function and a description of the transformation, write the equation of the transformed function $f(x)$.

a. **Quadratic - expanded horizontally by a factor of 2, translated 7 units up.**

$$f(x) = \frac{1}{2}x^2 + 7$$

b. **Cubic - reflected over the x axis and translated 9 units down.**

$$f(x) = -x^3 - 9$$

c. **Absolute value - translated 3 units up, translated 8 units right.**

$$f(x) = |x-8| + 3$$

d. **Reciprocal - translated 1 unit up.**

$$f(x) = \frac{1}{x} + 1$$

Sample Problem 3: Use the graph of parent function to graph each function. Find the domain and the range of the new function.

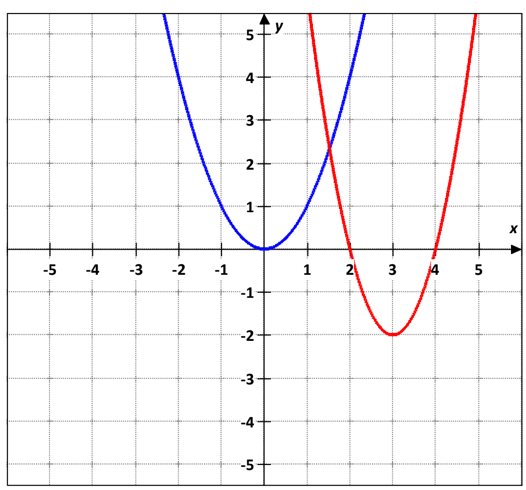
a. $h(x) = 2(x-3)^2 - 2$

$h(x) = 2(x-3)^2 - 2$ →

Parent function $f(x) = x^2$ →

Transformation:
Expand vertically by a factor of 2
Translated 2 units down
Translated 3 units right

$D = (-\infty, \infty)$
$R = (-2, \infty)$



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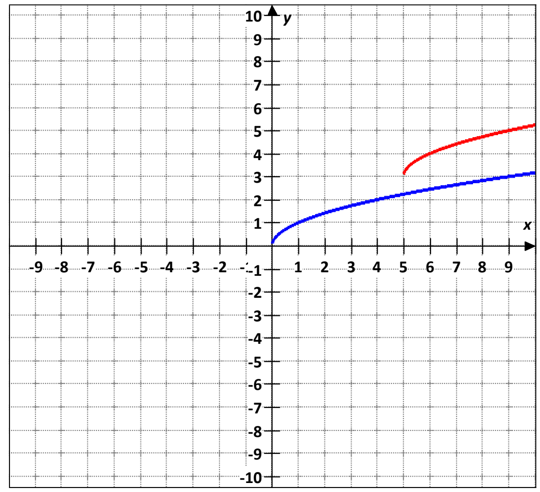
Parent Functions and Transformations Guided Notes

b. $h(x) = \sqrt{x-5} + 3$

$h(x) = \sqrt{x-5} + 3$ →

Parent function $f(x) = \sqrt{x}$ →



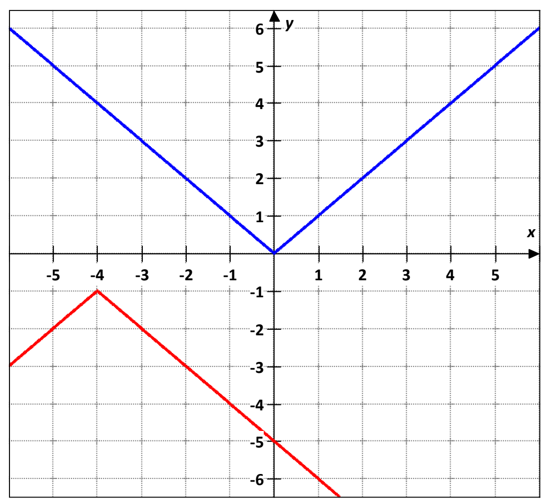
Transformation:
Translated 3 units up
Translated 5 units right

$D = [5.\infty)$
$R = (3, \infty)$

c. $h(x) = -|x+4| - 1$

$h(x) = -|x+4| - 1$ →

Parent function $f(x) = |x|$ →



Transformation:
Reflected in the x axis
Translated 1 unit down
Translated 4 units left

$D = (-\infty. \infty)$
$R = (-\infty, -1]$

Transformations with Absolute Value

$h(x) = |f(x)|$

This transformation reflects any portion of the graph of $f(x)$ that is below the x -axis so that it is above the x -axis.

$h(x) = f(|x|)$

This transformation results, in the portion of the graph of $f(x)$ that is to the left of the y-axis, being replaced by a reflection of the portion to the right of the y -axis.

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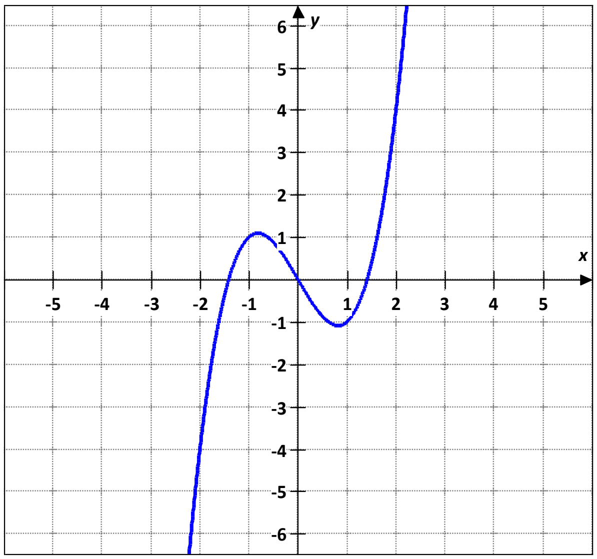
Parent Functions and Transformations Guided Notes

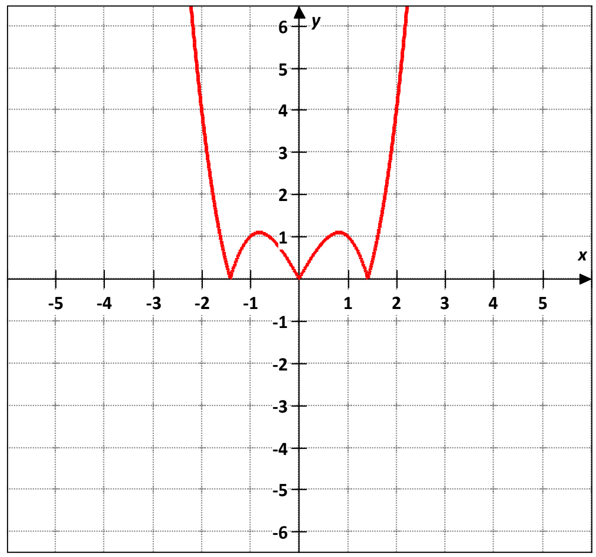
Sample Problem 4: **Graph each function.**

a. $f(x) = x^3 - 2x$ **Graph** $h(x) = |x^3 - 2x|$

$f(x) = x^3 - 2x$ →

$h(x) = |x^3 - 2x|$ →

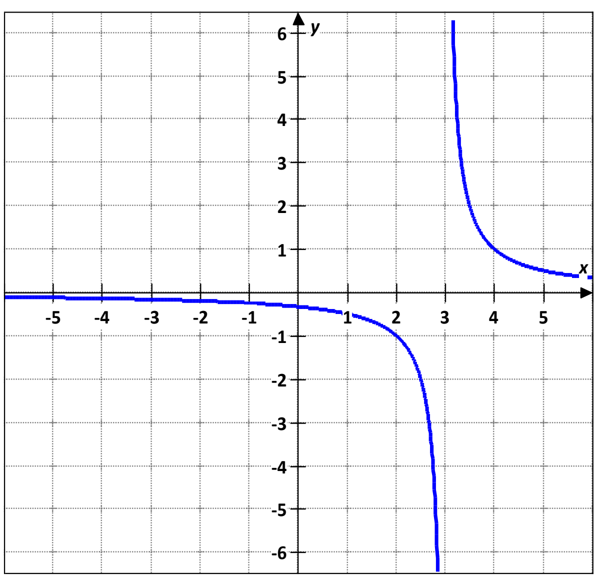


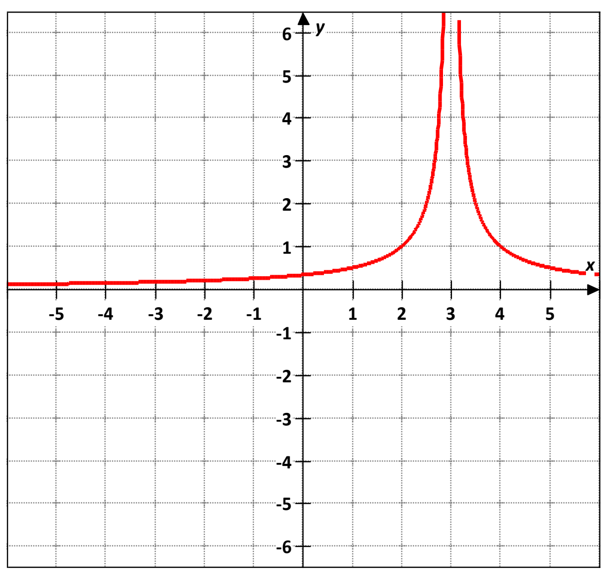


b. $f(x) = \dfrac{1}{x-3}$ **Graph** $h(x) = \dfrac{1}{|x-3|}$

$f(x) = \frac{1}{x-3}$ →

$h(x) = \frac{1}{|x-3|}$ →





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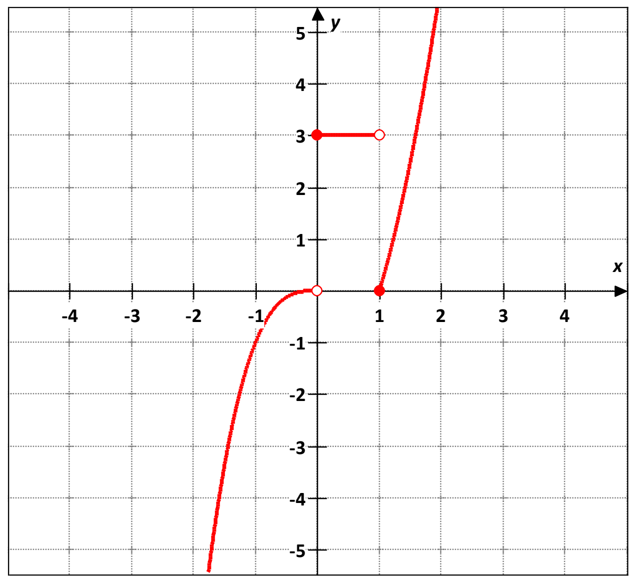
Parent Functions and Transformations Guided Notes

Graph a Piecewise-Defined Function

Sample Problem 5: **Graph each piecewise function.**

a.

$$f(x) = \begin{cases} -x^3 & \text{if } x < 0 \\ 3 & \text{if } 0 \le x < 1 \\ 2x^2 - 2 & \text{if } x \ge 1 \end{cases}$$



b.

$$f(x) = \begin{cases} 3x^2 & \text{if } x \le -1 \\ -2 & \text{if } -1 < x < 2 \\ |x^2 - 1| & \text{if } x \ge 2 \end{cases}$$