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Rational Numbers

Unit 1 Lesson 4

Students will be able to:

- Define rational numbers.
- Determine whether a number is rational or irrational.
- Change fractions to decimals.
- Determine whether a decimal number is terminating or non-terminating, repeating or non-repeating.
- Change terminating decimals to fractions.
- Change non-terminating and repeating decimals to fractions.

Key Vocabulary:

Rational Numbers

Irrational Numbers

Terminating Decimal

Non-terminating Decimal

Repeating Decimal

Non-repeating Decimal

Fractions

RATIONAL NUMBERS

Rational Numbers are numbers that can be written in the form:

$\frac{\boldsymbol{a}}{\boldsymbol{b}}$ where $\boldsymbol{a}$ and $\boldsymbol{b}$ are integers, and $\boldsymbol{b}$ is not equal to $\mathbf{0}$ $(\boldsymbol{b} \neq \mathbf{0})$

It is very important to remember that dividing a number by zero is not allowed as the answer is **undefined**.

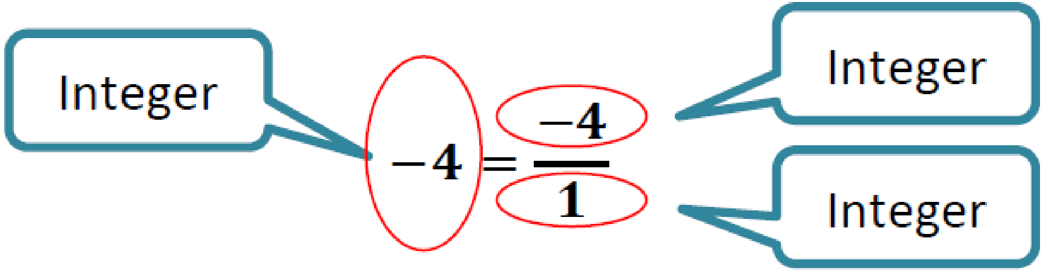
$$\frac{5}{0} = undefined$$

RATIONAL NUMBERS

Integers are rational numbers. Every integer can be written in the form $\frac{a}{b}$***:***

Examples:

-4 can be written as $\frac{-4}{1}$



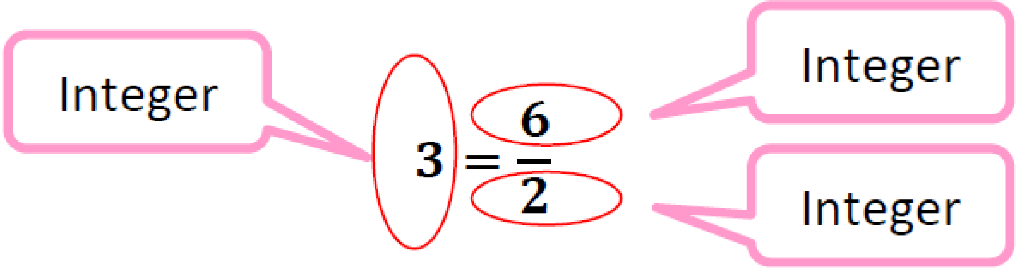
$$-4 = \frac{-4}{1}$$

RATIONAL NUMBERS

Integers are rational numbers. Every integer can be written in the form $\frac{a}{b}$:

Examples:

3 can be written as $\frac{6}{2}$



RATIONAL NUMBERS

Non-integers can be rational numbers too. Some of them can also be written in the form $\frac{a}{b}$:

Examples:

$\mathbf{0.25}$ can be written as $\frac{1}{4}$

$$0.25 = \frac{1}{4}$$

Non-Integer (0.25)

Integer (1)

Integer (4)

RATIONAL NUMBERS

Non-integers can be rational numbers too. Some of them can also be written in the form $\frac{a}{b}$:

Examples:

$\mathbf{0.42}$ can be written as $\frac{42}{100}$

Non-Integer

$$0.42 = \frac{42}{100}$$

Integer

Integer

RATIONAL NUMBERS

This only shows that a **rational number** can be expressed as a **fraction** or **decimal**. It can be transformed into its equivalent fraction or decimal form, and vice versa.

Study the examples on the next slide.

RATIONAL NUMBERS

Rational Numbers Expressed as Fractions			Rational Numbers Expressed as Decimals		
-5	$=$	$\frac{-15}{3}$	$\frac{-1}{2}$	$=$	-0.5
7	$=$	$\frac{7}{1}$	$\frac{3}{4}$	$=$	0.75
2	$=$	$\frac{-2}{-1}$	$\frac{5}{100}$	$=$	0.05

RATIONAL NUMBERS

When rational numbers expressed as fractions are divided, the quotient can be a terminating or repeating decimal number.

A **terminating decimal number** is one that has a finite number of digits. This means that the digits end.

TERMINATING DECIMAL NUMBER
$\frac{3}{4} = 0.75$
$\frac{13}{25} = 0.52$

RATIONAL NUMBERS

When rational numbers expressed as fractions are divided, the quotient can be a terminating or repeating decimal number.

A **repeating decimal number** is one that has digits that repeats and never ends.

REPEATING DECIMAL NUMBER
$\frac{1}{3} = 0.333333333333 \ldots$ or $0.\overline{3}$
$\frac{4}{11} = 0.363636363636 \ldots$ or $0.\overline{36}$

IRRATIONAL NUMBERS

On the other hand, there are numbers that are not rational. They are called **irrational numbers**. Unlike rational numbers, irrational numbers when expressed in decimal form can be a **non-terminating** and **non-repeating**.

A **non-terminating** and **non-repeating** decimal number has digits that don't repeat and goes on and on without end.

IRRATIONAL NUMBERS

Examples:

$\sqrt{2}$ can't be written in the form $\frac{a}{b}$.

$\sqrt{5}$ can't be written in the form $\frac{a}{b}$.

$\sqrt{7}$ can't be written in the form $\frac{a}{b}$.

NON-TERMINATING AND NON REPEATING DECIMAL NUMBER
$\sqrt{2} = 1.414213562 \ldots$
$\pi = 3.141592654 \ldots$

Sample Problem 1: Tell whether each of the following are rational or irrational numbers.

a. -9

b. $\frac{1}{2}$

c. $0.45454545\ldots$

d. $\sqrt{7}$

e. $\frac{10}{30}$

f. $\frac{2}{3}$

Sample Problem 1: Tell whether each of the following are rational or irrational numbers.

g. -1

h. 24

i. $\frac{6}{7}$

j. $\sqrt{25}$

Sample Problem 1: Solution

a. -9 RATIONAL

b. $\frac{1}{2}$ RATIONAL

c. $0.45454545\ldots$ RATIONAL

d. $\sqrt{7}$ IRRATIONAL

e. $\frac{10}{30}$ RATIONAL

f. $\frac{2}{3}$ RATIONAL

Sample Problem 1: Solution

g. -1 RATIONAL

h. 24 RATIONAL

i. $\frac{6}{7}$ RATIONAL

j. $\sqrt{25}$ RATIONAL

CHANGING FRACTIONS TO DECIMALS

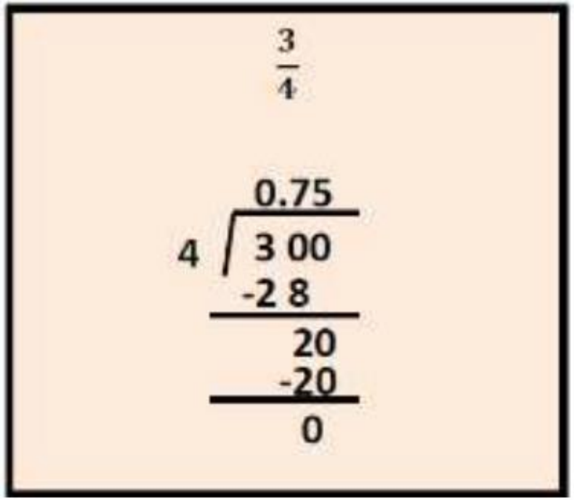
As mentioned earlier, **rational numbers can be in fraction or decimal form**. These forms are equivalent and can be transformed into one form or another. Changing fractions into its decimal form is like doing a simple division.

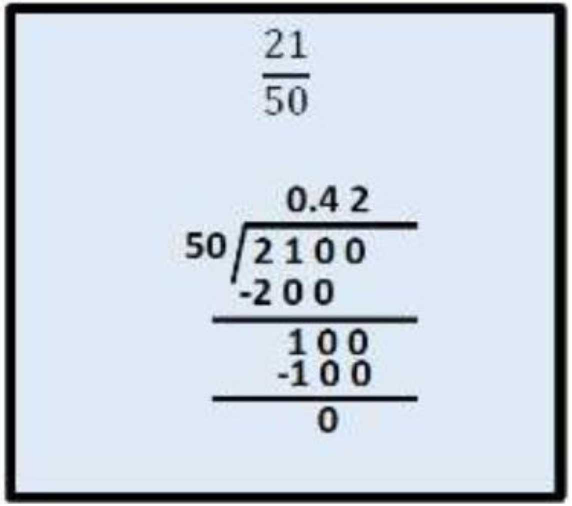
The required step is very simple:

Divide the numerator by the denominator.

CHANGING FRACTIONS TO DECIMALS

Example 1:

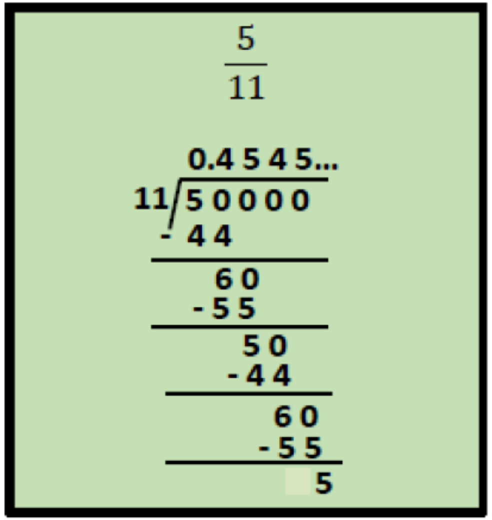


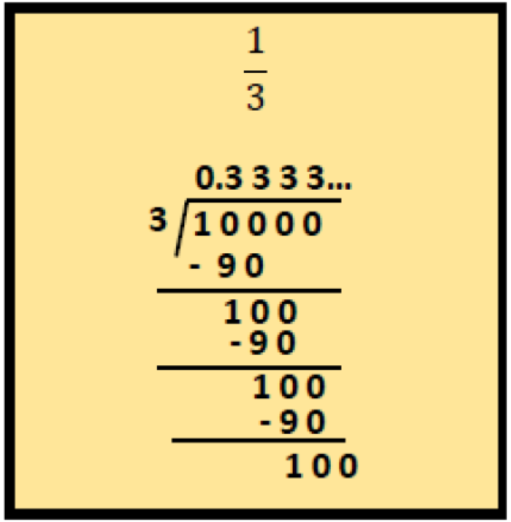


In the examples above, the rational numbers expressed as decimal numbers are **terminating decimals**.

CHANGING FRACTIONS TO DECIMALS

Example 2:





In the examples above, the rational numbers expressed as decimal numbers are **repeating and non-terminating decimals**. These types of decimals repeat over and over again, without end.

CHANGING FRACTIONS TO DECIMALS

***Repeating** and **non-terminating decimal numbers** can be written in two different ways:*

$\frac{5}{11} = 0.4545\ldots$	$\frac{5}{11} = 0.\overline{45}$
The three dots called **ellipses** indicates that 45 is repeated indefinitely.	A bar, also called ***vinculum***, can be placed on top of the digit/s being repeated.

Therefore, $\frac{1}{3} = 0.333\ldots$ **or** $0.\overline{3}$

CHANGING FRACTIONS TO DECIMALS

WARNING: Sometimes, it takes several decimals to know whether the equivalent decimal of a given fraction is a repeating decimal.

Example: $\frac{6}{7} = 0.857142857142857142857142\ldots$

Sample Problem 2: Express the following fractions as decimals and determine whether the decimal number terminates or repeats.

a. $\frac{5}{8}$

b. $\frac{2}{5}$

Sample Problem 2: Express the following fractions as decimals and determine whether the decimal number terminates or repeats.

c. $\frac{3}{25}$

d. $\frac{12}{15}$

Sample Problem 2: Express the following fractions as decimals and determine whether the decimal number terminates or repeats.

e. $\frac{5}{6}$

f. $\frac{13}{9}$

Sample Problem 2: Express the following fractions as decimals and determine whether the decimal number terminates or repeats.

g. $\frac{11}{15}$

h. $\frac{9}{11}$

Sample Problem 2: Solution

a. $\frac{5}{8}$ Terminating and non-repeating

```
    0.625
  ________
8 ) 5000
 -  48
  ________
     20
    -16
  ________
      40
     -40
  ________
       0
```

b. $\frac{2}{5}$ Terminating and non-repeating

```
    0.4
  _____
5 ) 20
 -  20
  _____
     0
```

Sample Problem 2: Solution

c. $\frac{3}{25}$ Terminating and non-repeating

$$
\begin{array}{r}
0.12 \\
25\overline{)\,3000} \\
-\ 25 \\
\hline
50 \\
-50 \\
\hline
0
\end{array}
$$

d. $\frac{12}{15}$ Terminating and non-repeating

$$
\begin{array}{r}
0.8 \\
15\overline{)\,120} \\
-\ 120 \\
\hline
0
\end{array}
$$

Sample Problem 2: Solution

e. $\frac{5}{6}$ Non-terminating and repeating

```
   0.8 3 3
6 /5 0 0 0
 - 4 8
  -------
    2 0
   -1 8
  -------
      2 0
     -1 8
  -------
        2
```

f. $\frac{13}{9}$ Non-terminating and repeating

```
    1.4 4...
9 /1 3 0 0
 -  9
  -------
    4 0
   -3 6
  -------
      4 0
     -3 6
  -------
        4
```

Sample Problem 2: Solution

g. $\frac{11}{15}$ Non-terminating and repeating

$$
\begin{array}{r}
0.733\ldots \\
15\,\overline{)11000} \\
-\ 105 \\
\hline
50 \\
-45 \\
\hline
50 \\
-45 \\
\hline
5
\end{array}
$$

h. $\frac{9}{11}$ Non-terminating and repeating

$$
\begin{array}{r}
0.8181\ldots \\
11\,\overline{)90000} \\
-\ 88 \\
\hline
20 \\
-11 \\
\hline
90 \\
-88 \\
\hline
20 \\
-11 \\
\hline
9
\end{array}
$$

CHANGING TERMINATING DECIMALS TO FRACTIONS

Let's now reverse the process. This time, we'll transform terminating decimals to its equivalent fraction. In changing decimal numbers to fractions, we use its digits (without the decimal point) as the numerator. The denominator on the other hand must be a power of 10.

We need to select the appropriate denominator such as 10, 100, 1 000, 10 000, and so on. Don't forget to always change the fraction in its lowest term, if possible.

CHANGING TERMINATING DECIMALS TO FRACTIONS

Examples:

"**0.5**" is read as **5 tenths**.

It is written as: $\frac{5}{10}$

It is simplified as: $\frac{1}{2}$

$$\mathbf{0.5 = \frac{5}{10}\ \mathit{or}\ \frac{1}{2}}$$

"**0.12**" is read as **12 hundredths**.

It is written as: $\frac{12}{100}$

It is simplified as: $\frac{3}{25}$

$$\mathbf{0.12 = \frac{12}{100}\ \mathit{or}\ \frac{3}{25}}$$

CHANGING TERMINATING DECIMALS TO FRACTIONS

Examples:

"**0.25**" is read as **25 hundredths.**

It is written as: $\frac{25}{100}$

It is simplified as: $\frac{1}{4}$

$$0.25 = \frac{25}{100} \text{ or } \frac{1}{4}$$

"**0.625**" is read as **625 thousandths.**

It is written as: $\frac{625}{1000}$

It is simplified as: $\frac{5}{8}$

$$0.625 = \frac{625}{1000} \text{ or } \frac{5}{8}$$

Sample Problem 3: Express the following terminating decimals in its equivalent fraction.

a. 0.9

b. 0.24

c. 0.125

d. 0.375

Sample Problem 3: Solution

a. $\mathbf{0.9 = \frac{9}{10}}$

b. $\mathbf{0.24 = \frac{24}{100}}$ $\boldsymbol{or}$ $\mathbf{\frac{6}{25}}$

c. $\mathbf{0.125 = \frac{125}{1000}}$ $\boldsymbol{or}$ $\mathbf{\frac{1}{8}}$

d. $\mathbf{0.375 = \frac{375}{1000}}$ $\boldsymbol{or}$ $\mathbf{\frac{3}{8}}$

CHANGING REPEATING AND NON-TERMINATING DECIMALS TO FRACTIONS

While changing terminating decimal numbers to fractions is a piece of cake, doing the same for repeating and non-terminating decimal numbers is a different story. But there is nothing to worry about. The steps on how to do it are explained clearly in the examples below.

Example 1: Change **0.888…** to its fraction form.

CHANGING REPEATING AND NON-TERMINATING DECIMALS TO FRACTIONS

Step 1: Let's create an equation by choosing any variable. We could let x be equal to the given repeating and non-terminating decimal number.

$$\boldsymbol{x = 0.888\ldots}$$

CHANGING REPEATING AND NON-TERMINATING DECIMALS TO FRACTIONS

Step 2: Examine the repeating decimal closely to determine the number of digit or digits that repeats.

$$x = 0.888\ldots$$

There is only one repeating digit, 8.

CHANGING REPEATING AND NON-TERMINATING DECIMALS TO FRACTIONS

Step 3: Using the original equation, we need to place the repeating digit to the left of the decimal point. Since we only have one repeating digit, we'll multiply both sides of the original equation by 10.

$$(10)x = 0.888\ldots(10)$$

$$10x = 8.888\ldots$$

CHANGING REPEATING AND NON-TERMINATING DECIMALS TO FRACTIONS

Step 4: Subtract the two equations.

$$10x = 8.888\ldots$$

$$x = 0.888\ldots$$

$$9x = 8$$

CHANGING REPEATING AND NON-TERMINATING DECIMALS TO FRACTIONS

Step 5: Solve for x by dividing both sides by 9.

$$\frac{9x}{9} = \frac{8}{9}$$

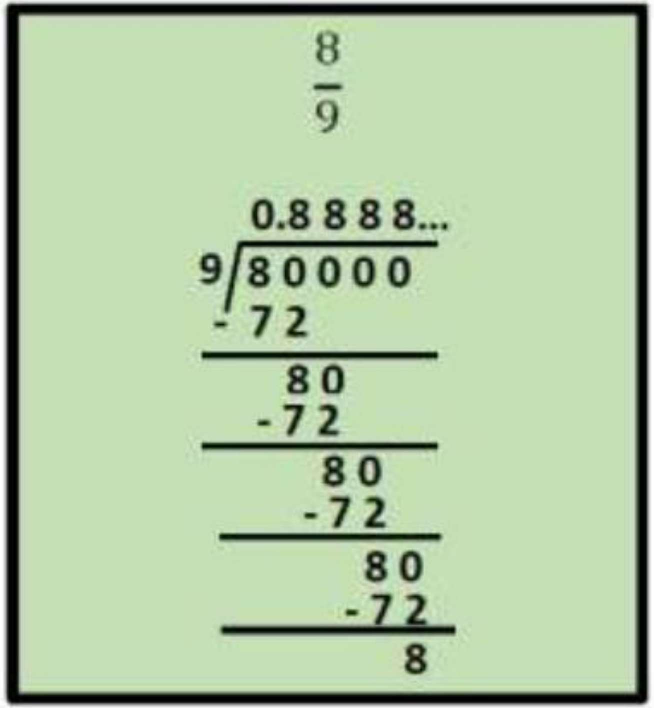
$$x = \frac{8}{9}$$

So, $\mathbf{0.888\ldots = \frac{8}{9}}$. We can't simplify $\mathbf{\frac{8}{9}}$ any further.

CHANGING REPEATING AND NON-TERMINATING DECIMALS TO FRACTIONS

Let’s confirm if we are correct.

Or better yet, you may use a calculator to confirm.



CHANGING REPEATING AND NON-TERMINATING DECIMALS TO FRACTIONS

Example 2: Change $0.\overline{45}$ to its fraction form.

Step 1: Let's create an equation by choosing any variable. We could let x be equal to the given repeating and non-terminating decimal number.

$$\boldsymbol{x = 0.\overline{45}}$$

CHANGING REPEATING AND NON-TERMINATING DECIMALS TO FRACTIONS

Step 2: Examine the repeating decimal closely to determine the number of digit or digits that repeats.

$$x = 0.\overline{45}$$

There are two repeating digits, 45.

CHANGING REPEATING AND NON-TERMINATING DECIMALS TO FRACTIONS

Step 3: Using the original equation, we need to place the repeating digit to the left of the decimal point. Since we have two repeating digits, we'll multiply both sides of the original equation by 100.

$$(100)x = 0.\overline{45}(100)$$

$$100x = 45.\overline{45}$$

CHANGING REPEATING AND NON-TERMINATING DECIMALS TO FRACTIONS

Step 4: Subtract the two equations.

$$100x = 45.\overline{45}$$

$$x = 0.\overline{45}$$

$$99x = 45$$

CHANGING REPEATING AND NON-TERMINATING DECIMALS TO FRACTIONS

Step 5: Solve for x by dividing both sides by 99.

$$\frac{99x}{99} = \frac{45}{99}$$

$$x = \frac{45}{99}$$

So, $0.\overline{45} = \frac{45}{99}$ **or** $\frac{5}{11}$.

Sample Problem 4: Express the following terminating decimals in its equivalent fraction.

a. $\mathbf{0.222\ldots}$

b. $\mathbf{0.2\overline{7}}$

Sample Problem 4: Express the following terminating decimals in its equivalent fraction.

c. $\mathbf{0.\overline{125}}$

d. $\mathbf{3.\overline{124}}$

Sample Problem 4: Solution

a. $0.222\ldots$

$$x = 0.222\ldots$$

$$(10)x = 0.222\ldots(10)$$
$$10x = 2.222$$

$$10x = 2.222$$
$$x = 0.222\ldots$$
$$9x = 2$$

$$\frac{9x}{9} = \frac{2}{9}$$
$$x = \frac{2}{9}$$

$$0.222\ldots = \frac{2}{9}$$

b. $0.2\overline{7}$

$$x = 0.2\overline{7}$$
$$(10)x = 0.2\overline{7}\ldots(10)$$
$$10x = 2.\overline{7}$$

$$(100)x = 0.2\overline{7}\ldots(100)$$
$$100x = 27.\overline{7}$$

$$100x = 27.\overline{7}$$
$$10x = 2.\overline{7}$$
$$90x = 25$$

$$\frac{90x}{90} = \frac{25}{90}$$
$$x = \frac{25}{90}$$

$$0.2\overline{7} = \frac{25}{90}\ or\ \frac{5}{18}$$

Sample Problem 4: Solution

c. $0.\overline{125}$

$$x = 0.\overline{125}$$

$$(1000)x = 0.\overline{125}(1000)$$
$$1000x = 125.\overline{125}$$

$$\begin{array}{r} 1000x = 125.\overline{125} \\ x = 0.\overline{125} \\ \hline 999x = 125 \end{array}$$

$$\frac{999x}{999} = \frac{125}{999}$$
$$x = \frac{125}{999}$$

$$0.\overline{125} = \frac{125}{999}$$

d. $3.\overline{124}$

$$x = 3.\overline{124}$$

$$(1000)x = 3.\overline{124}(1000)$$
$$1000x = 3124.\overline{124}$$

$$\begin{array}{r} 1000x = 3124.\overline{124} \\ x = 3.\overline{124} \\ \hline 999x = 3121 \end{array}$$

$$\frac{999x}{999} = \frac{3121}{999}$$
$$x = \frac{3121}{999}$$

$$3.\overline{124} = \frac{3121}{999} \text{ or } 3\frac{124}{999}$$