**SQUARE OF A NUMBER**

To square a number means to multiply a number by itself.

This small number called “**exponent**” indicates that the base, which is 5, will be a factor twice.

This is read as “5 squared is 25”.

25 is obtained by making 5 a factor, twice.

**More examples:**

**0 squared**

**1 squared**

**-1 squared**

**2 squared**

**-2 squared**

**3 squared**

**-3 squared**

**are called PERFECT SQUARE NUMBERS.**

Relating this to shapes: To create a perfect square, all four sides must have equal measures.

1

2

3

4

2

2

**2 squared**

**Sample Problem 1:** Read and answer the questions below:

1. Is 81 a perfect square number? Justify your answer.

**Answer:**

81 is a perfect square number because it can be obtained by making 9 a factor twice.

1. Is 121 a perfect square number? Justify your answer.

**Answer:**

121 is a perfect square number because it can be obtained by making 11 a factor twice.

**Sample Problem 2:** Square the following numbers.

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| --- | --- |
| a) 7 | b) -5 |
| c) 8 | d) |

**SQUARE ROOT OF A NUMBER**

Subtraction is the inverse of addition, and so is division the inverse of multiplication. The inverse of squaring a number is by getting its **SQUARE ROOT**.

The symbol of square root is denoted by the symbol: . The symbol is also referred to as the “**radical**” sign. Any number placed inside the radical sign is called a “**radicand**”.

**If , then a number is a square root of ,**

All positive real numbers have two roots, one positive and one negative. Normally, if we want to indicate that we want to get both roots, we can use the symbol: .

**Study the examples below:**

**Since and , then 1 and -1 are both square roots of 1.**

**Since and , then 2 and -2 are both square roots of 4.**

**Since and , then 3 and -3 are both square roots of 9.**

**can also be written as**

**PRINCIPAL SQUARE ROOT**

The positive square root of any non-negative real number is called the **principal square root**.

**For any non-negative real numbers and such that is called the principal square root of , denoted by .**

**Since**

The principal square root of 1 is 1.

**Since**

The principal square root of 4 is 2.

**Since**

The principal square root of 9 is 3.

**The square root of a negative number is not defined. This means that it has no value at all.**

**Sample Problem 3:** Find the value of the following.

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| --- | --- |
| a) | b) undefined |
| c) | d) |
| e) | f) |

**APPROXIMATING THE SQUARE ROOT OF A NUMBER**

The square root of perfect square numbers like 1, 4, 9, 16, 25, and 36 are rational numbers. Non-perfect square numbers on the other hand have irrational numbers as roots. By irrational, it means that the square roots cannot be expressed as a ratio of two integers.

5 can be expressed as a ratio of two integers:

It can’t be expressed as a ratio of two integers:

Irrational numbers complete the set of real numbers. They fill up the spaces between rational numbers. This can be visualized using a number line.



Look at the approximate square roots of some non-perfect square numbers.

The square root of a non-perfect square number can be found between two rational numbers. It can be approximated by looking for two consecutive integers or rational numbers where the square root lies in between.



is found between rational numbers and .

is found between rational numbers and .

is found between rational numbers and .

**Example 1:** Find two consecutive integers between which lies.

**Step 1:** Examine the radicand.

The radicand is 12.

**Step 2:** Look for a perfect square number, less than and closest to the radicand.

The closest perfect square number less than 12 is 9.

**Step 3:** Look for a perfect square number, greater than and closest to the radicand.

The closest perfect square number greater than 12 is 16.

**Therefore, the value of is between rational numbers 3 and 4.**

**Grab a calculator and confirm to confirm the answer:**



**…**

**is between 3 and 4.**

**Example 2:** Find two rational numbers with two decimal places between which lies.

**Step 1:** Examine the radicand.

The radicand is 7.

**Step 2:** Look for a perfect square number, less than and closest to the radicand.

The closest perfect square number less than 7 is 4.

**Step 3:** Look for a perfect square number, greater than and closest to the radicand.

The closest perfect square number greater than 7 is 9.

**Therefore, the value of is between rational numbers 2 and 3.**

To find two rational numbers with two decimal places between which lies, let’s do some estimation. Since liesbetween 2 and 3, we can square the decimal numbers between 2 and 3 to start the estimation.

**So, lies between 2.6 and 2.7.**

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To find the closest approximate square root, let’s continue the estimation. Since we are looking for two rational numbers with two decimal places between which lies, we’ll use the decimal numbers between 2.6 and 2.7.

**So, lies between 2.64 and 2.65.**

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**So, the value of is between rational numbers with two decimal places 2.64 and 2.65.**

**Example 3:** Find two rational numbers with two decimal places between which lies.

*(\*Note: follow the same steps as we did in the previous examples.)*

Since 18 is between 16 and 25, must be between and

By estimation, we have:

**So, lies between 4.2 and 4.3.**

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To find the two rational numbers with two decimal places between which lies, let’s estimate further:

**So, lies between 4.24 and 4.25.**

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**So, the value of is between rational numbers with two decimal places 4.24 and 4.25.**

**Sample Problem 4:** Find two consecutive integers between which the given square root lies.

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| --- | --- |
| a)  The radicand is 11.  The closest perfect square number  less than 11 is 9.  The closest perfect square number  greater than 11 is 16.  The square root of is between 3 and 4. | b)  The radicand is 35.  The closest perfect square number  less than 35 is 25.  The closest perfect square number  greater than 35 is 36.  The square root of is between 5 and 6. |
| c)  The radicand is 28.  The closest perfect square number  less than 28 is 25.  The closest perfect square number  greater than 28 is 16.  The square root of is between 5 and 6. | d)  The radicand is 91.  The closest perfect square number  less than 91 is 81.  The closest perfect square number  greater than 91 is100.  The square root of is between 9 and 10. |

**Sample Problem 5:** Determine two rational numbers with two decimal places between which the given square root lies.

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a)  Since 5 is between 4 and 9, must be between and  By estimation, we have:  **So, lies between 2.2 and 2.3.**   |  | | --- | |  | |  | |  |   To find the two rational numbers with two decimal places between which lies, let’s estimate further:  **So, lies between 2.23 and 2.24.**   |  | | --- | |  | |  | |  | |  |   The square root of is between  2.23 and 2.24. | b)  Since 23 is between 16 and 25, must be between and  By estimation, we have:  **So, lies between 4.7 and 4.8.**   |  | | --- | |  | |  | |  | |  |   To find the two rational numbers with two decimal places between which lies, let’s estimate further:  **So, lies between 4.79 and 4.80.**   |  | | --- | |  | |  | |  | |  | |  | |  |   The square root of is between  4.79 and 4.80. |

**APPROXIMATING THE SQUARE ROOT OF A NUMBER BY AVERAGING**

Another strategy to approximate square roots is by averaging. This is a less complicated method that involves square root estimates. Study the examples below:

**Example 1:** Approximate .

Since 12 is between 9 and 16, must be between and

**Step 1:** To find the first estimate, choose the integer that is closest to .

The integer closest to is 3.

The first estimate is 3.

**Step 2:** Divide the radicand by the first estimate.

**Step 3:** To find the second estimate, find the average of the quotient in Step 2 and the first estimate.

The second estimate is 3.5.

**Step 4:** Repeat Step 2. But this time, divide the radicand by the second estimate.

(Rounded to the nearest thousandths)

**Step 5:** To find the third estimate, repeat Step 3. This time, find the average of the quotient in Step 4 and the second estimate.

Therefore, the closest approximate of is 3.46.

**Example 2:** Approximate .

Since 5 is between 4 and 9, must be between and

**Step 1:** To find the first estimate, choose the integer that is closest to .

The integer closest to is 2.

The first estimate is 2.

**Step 2:** Divide the radicand by the first estimate.

**Step 3:** To find the second estimate, find the average of the quotient in Step 2 and the first estimate.

The second estimate is 2.25.

**Step 4:** Repeat Step 2. But this time, divide the radicand by the second estimate.

(Rounded to the nearest thousandths)

**Step 5:** To find the third estimate, repeat Step 3. This time, find the average of the quotient in Step 4 and the second estimate.

Therefore, the closest approximate of is 2.236.

**Example 3:** Approximate .

Since 24 is between 16 and 25, must be between and

**Step 1:** To find the first estimate, choose the integer that is closest to .

The integer closest to is 5.

The first estimate is 5.

**Step 2:** Divide the radicand by the first estimate.

**Step 3:** To find the second estimate, find the average of the quotient in Step 2 and the first estimate.

The second estimate is 4.9.

**Step 4:** Repeat Step 2. But this time, divide the radicand by the second estimate.

(Rounded to the nearest thousandths)

**Step 5:** To find the third estimate, repeat Step 3. This time, find the average of the quotient in Step 4 and the second estimate.

Therefore, the closest approximate of is 4.899.

**Sample Problem 6:** Approximate the square root up to the third estimate by averaging

|  |  |
| --- | --- |
| a)  Since 8 is between 4 and 9, must be between and  Step 1: The integer closest to is 3.  The first estimate is 3.  Step 2: Divide the radicand by the first estimate.  Step 3: To find the second estimate, find the average of the quotient in Step 2 and the first estimate.  The second estimate is 4.832.  Step 4: Repeat Step 2. But this time, divide the radicand by the second estimate.  Step 5: To find the third estimate, repeat Step 3. This time, find the average of the quotient in Step 4 and the second estimate.  The closest approximate of is 2.83 | b)  Since 29 is between 25 and 36, must be between and  Step 1: The integer closest to is 5.  The first estimate is 5.  Step 2: Divide the radicand by the first estimate.  Step 3: To find the second estimate, find the average of the quotient in Step 2 and the first estimate.  The second estimate is 5.4.  Step 4: Repeat Step 2. But this time, divide the radicand by the second estimate.  Step 5: To find the third estimate, repeat Step 3. This time, find the average of the quotient in Step 4 and the second estimate.  The closest approximate of is 5.385. |