**SQUARE OF A NUMBER**

To square a number means to multiply a number by itself.

This small number called “**exponent**” indicates that the base, which is 5, will be a factor twice.

This is read as “5 squared is 25”.

25 is obtained by making 5 a factor, twice.

**More examples:**

**0 squared** $=0^{2}=0∙0=0$

**1 squared** $=1^{2}=1∙1=1$

**-1 squared** $=(-1)^{2}=-1∙-1=1$

**2 squared** $=2^{2}=2∙2=4$

**-2 squared** $=(-2)^{2}=-2∙-2=4$

**3 squared** $=3^{2}=3∙3=9$

**-3 squared** $=(-3)^{2}=-3∙-3=9$

$0, 1, 4, 9,…$ **are called PERFECT SQUARE NUMBERS.**

Relating this to shapes: To create a perfect square, all four sides must have equal measures.

1

2

3

4

2

2

**2 squared**

**Sample Problem 1:** Read and answer the questions below:

1. Is 81 a perfect square number? Justify your answer.

**Answer:**

81 is a perfect square number because it can be obtained by making 9 a factor twice.

1. Is 121 a perfect square number? Justify your answer.

**Answer:**

121 is a perfect square number because it can be obtained by making 11 a factor twice.

**Sample Problem 2:** Square the following numbers.

|  |  |
| --- | --- |
| a) 7$$7^{2}=7∙7=49$$ | b) -5$$(-5)^{2}=-5∙-5=25$$ |
| c) 8$$8^{2}=8∙8=64$$ | d) $-10$$$(-10)^{2}=(-10)∙(-10)=100$$ |

**SQUARE ROOT OF A NUMBER**

Subtraction is the inverse of addition, and so is division the inverse of multiplication. The inverse of squaring a number is by getting its **SQUARE ROOT**.

The symbol of square root is denoted by the symbol: $\sqrt{ }$. The symbol is also referred to as the “**radical**” sign. Any number placed inside the radical sign is called a “**radicand**”.

 **If** $a^{2}=b$**, then a number** $a$ **is a square root of** $b$**,** $a=\sqrt{b}.$

All positive real numbers have two roots, one positive and one negative. Normally, if we want to indicate that we want to get both roots, we can use the symbol: $\pm \sqrt{ }$.

**Study the examples below:**

**Since and , then 1 and -1 are both square roots of 1.**

**Since and , then 2 and -2 are both square roots of 4.**

**Since and , then 3 and -3 are both square roots of 9.**

$+1 and-1, +2 and-2, +3 and-3$ **can also be written as** $\pm 1, \pm 2 and \pm 3.$

**PRINCIPAL SQUARE ROOT**

The positive square root of any non-negative real number is called the **principal square root**.

**For any non-negative real numbers** $a$ **and** $b$ **such that** $a^{2}=b,$$a $**is called the principal square root of** $b$**, denoted by** $\sqrt{b}$**.**

**Since** $1^{2}=1, then \sqrt{1}=1 $

The principal square root of 1 is 1.

**Since** $2^{2}=4, then \sqrt{4}=2 $

The principal square root of 4 is 2.

**Since** $3^{2}=9, then \sqrt{9}=3 $

The principal square root of 9 is 3.

**The square root of a negative number is not defined. This means that it has no value at all.**

$$\sqrt{-1}=undefined $$

$$\sqrt{-4}=undefined $$

$$\sqrt{-9}=undefined $$

**Sample Problem 3:** Find the value of the following.

|  |  |
| --- | --- |
| a) $\pm \sqrt{49}=\pm 7$ | b) $\sqrt{-49}$undefined  |
| c) $\sqrt{121}=11$ | d) $\pm \sqrt{16}=\pm 4 $ |
| e) $\sqrt{100}=10$ | f) $\pm \sqrt{64}=\pm 8$ |

**APPROXIMATING THE SQUARE ROOT OF A NUMBER**

The square root of perfect square numbers like 1, 4, 9, 16, 25, and 36 are rational numbers. Non-perfect square numbers on the other hand have irrational numbers as roots. By irrational, it means that the square roots cannot be expressed as a ratio of two integers.

5 can be expressed as a ratio of two integers:

It can’t be expressed as a ratio of two integers:

Irrational numbers complete the set of real numbers. They fill up the spaces between rational numbers. This can be visualized using a number line.

Look at the approximate square roots of some non-perfect square numbers.

$$\sqrt{2}=1.414…$$

$$\sqrt{8}=2.828…$$

$$\sqrt{10}=3.162…$$

The square root of a non-perfect square number can be found between two rational numbers. It can be approximated by looking for two consecutive integers or rational numbers where the square root lies in between.

$\sqrt{2}$ is found between rational numbers $1$and $2$.

$\sqrt{8}$ is found between rational numbers $2 $and $3$.

$\sqrt{10}$ is found between rational numbers $3 $and $4$.

**Example 1:** Find two consecutive integers between which $\sqrt{12}$ lies.

**Step 1:** Examine the radicand.

The radicand is 12.

**Step 2:** Look for a perfect square number, less than and closest to the radicand.

The closest perfect square number less than 12 is 9.

**Step 3:** Look for a perfect square number, greater than and closest to the radicand.

The closest perfect square number greater than 12 is 16.

$$\sqrt{9}<\sqrt{12}<\sqrt{16}$$

$$3<\sqrt{12}<4$$

**Therefore, the value of** $\sqrt{12}$ **is between rational numbers 3 and 4.**

**Grab a calculator and confirm to confirm the answer:**

**…**

$$\sqrt{12}≈3.464…$$

$3.464…$ **is between 3 and 4.**

**Example 2:** Find two rational numbers with two decimal places between which $\sqrt{7}$ lies.

**Step 1:** Examine the radicand.

The radicand is 7.

**Step 2:** Look for a perfect square number, less than and closest to the radicand.

The closest perfect square number less than 7 is 4.

**Step 3:** Look for a perfect square number, greater than and closest to the radicand.

The closest perfect square number greater than 7 is 9.

$$\sqrt{4}<\sqrt{7}<\sqrt{9}$$

$$2<\sqrt{7}<3$$

**Therefore, the value of** $\sqrt{7}$ **is between rational numbers 2 and 3.**

To find two rational numbers with two decimal places between which $\sqrt{7}$ lies, let’s do some estimation. Since $\sqrt{7}$liesbetween 2 and 3, we can square the decimal numbers between 2 and 3 to start the estimation.

**So, lies between 2.6 and 2.7.**

|  |
| --- |
| $$2.1^{2}=4.41$$ |
| $$2.2^{2}=4.84$$ |
| $$2.3^{2}=5.29$$ |
| $$2.4^{2}=5.76$$ |
| $$2.5^{2}=6.25$$ |
| $$2.6^{2}=6.76$$ |
| $$2.7^{2}=7.29$$ |

To find the closest approximate square root, let’s continue the estimation. Since we are looking for two rational numbers with two decimal places between which $\sqrt{7}$ lies, we’ll use the decimal numbers between 2.6 and 2.7.

**So, lies between 2.64 and 2.65.**

|  |
| --- |
| $$2.61^{2}=6.8121$$ |
| $$2.62^{2}=6.8644$$ |
| $$2.63^{2}=6.9619$$ |
| $$2.64^{2}=6.9696$$ |
| $$2.65^{2}=7.0225$$ |

**So, the value of** $\sqrt{7}$ **is between rational numbers with two decimal places 2.64 and 2.65.**

**Example 3:** Find two rational numbers with two decimal places between which $\sqrt{18}$ lies.

*(\*Note: follow the same steps as we did in the previous examples.)*

Since 18 is between 16 and 25, $\sqrt{18}$ must be between $\sqrt{16}$ and $\sqrt{25.}$

$$\sqrt{16}<\sqrt{18}<\sqrt{25}$$

$$4<\sqrt{18}<5$$

By estimation, we have:

**So, lies between 4.2 and 4.3.**

|  |
| --- |
| $$4.1^{2}=16.81$$ |
| $$4.2^{2}=17.64$$ |
| $$4.3^{2}=18.49$$ |

To find the two rational numbers with two decimal places between which $\sqrt{18}$ lies, let’s estimate further:

**So, lies between 4.24 and 4.25.**

|  |
| --- |
| $$4.21^{2}=17.7241$$ |
| $$4.22^{2}=17.8084$$ |
| $$4.23^{2}=17.8929$$ |
| $$4.24^{2}=17.9776$$ |
| $$4.25^{2}=18.0625$$ |

**So, the value of** $\sqrt{18}$ **is between rational numbers with two decimal places 4.24 and 4.25.**

**Sample Problem 4:** Find two consecutive integers between which the given square root lies.

|  |  |
| --- | --- |
| a) $\sqrt{11}$The radicand is 11.The closest perfect square numberless than 11 is 9.The closest perfect square numbergreater than 11 is 16.$$\sqrt{9}<\sqrt{11}<\sqrt{16}$$$$3<\sqrt{11}<4$$The square root of $\sqrt{11}$ is between 3 and 4. | b) $\sqrt{35}$The radicand is 35.The closest perfect square numberless than 35 is 25.The closest perfect square numbergreater than 35 is 36.$$\sqrt{25}<\sqrt{35}<\sqrt{36}$$$$5<\sqrt{35}<6$$The square root of $\sqrt{35}$ is between 5 and 6. |
| c) $\sqrt{28}$The radicand is 28.The closest perfect square numberless than 28 is 25.The closest perfect square numbergreater than 28 is 16.$$\sqrt{25}<\sqrt{28}<\sqrt{36}$$$$5<\sqrt{28}<6$$The square root of $\sqrt{28}$ is between 5 and 6. | d) $\sqrt{91}$The radicand is 91.The closest perfect square numberless than 91 is 81.The closest perfect square numbergreater than 91 is100.$$\sqrt{81}<\sqrt{91}<\sqrt{100}$$$$9<\sqrt{91}<10$$The square root of $\sqrt{91}$ is between 9 and 10. |

**Sample Problem 5:** Determine two rational numbers with two decimal places between which the given square root lies.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a) $\sqrt{5}$Since 5 is between 4 and 9, $\sqrt{5}$ must be between $\sqrt{4}$ and $\sqrt{9.}$$$\sqrt{4}<\sqrt{5}<\sqrt{9}$$$$2<\sqrt{5}<3$$By estimation, we have:**So, lies between 2.2 and 2.3.**

|  |
| --- |
| $$2.1^{2}=4.41$$ |
| $$2.2^{2}=4.48$$ |
| $$2.3^{2}=5.29$$ |

To find the two rational numbers with two decimal places between which $\sqrt{5}$ lies, let’s estimate further:**So, lies between 2.23 and 2.24.**

|  |
| --- |
| $$2.21^{2}=4.8841$$ |
| $$2.22^{2}=4.9284$$ |
| $$2.23^{2}=4.9729$$ |
| $$2.24^{2}=5.0176$$ |

The square root of $\sqrt{5}$ is between2.23 and 2.24. | b) $\sqrt{23}$Since 23 is between 16 and 25, $\sqrt{23}$ must be between $\sqrt{16}$ and $\sqrt{25.}$$$\sqrt{16}<\sqrt{23}<\sqrt{25}$$$$4<\sqrt{23}<5$$By estimation, we have:**So, lies between 4.7 and 4.8.**

|  |
| --- |
| $$4.6^{2}=21.16$$ |
| $$4.7^{2}=22.09$$ |
| $$4.8^{2}=23.04$$ |
|  |

To find the two rational numbers with two decimal places between which $\sqrt{23}$ lies, let’s estimate further:**So, lies between 4.79 and 4.80.**

|  |
| --- |
| $$4.76^{2}=22.6576$$ |
| $$4.77^{2}=22.7529$$ |
| $$4.78^{2}=22.8484$$ |
| $$4.79^{2}=22.9441$$ |
| $$4.80^{2}=23.04$$ |
|  |

The square root of $\sqrt{23}$ is between4.79 and 4.80. |

**APPROXIMATING THE SQUARE ROOT OF A NUMBER BY AVERAGING**

Another strategy to approximate square roots is by averaging. This is a less complicated method that involves square root estimates. Study the examples below:

**Example 1:** Approximate $\sqrt{12}$.

Since 12 is between 9 and 16, $\sqrt{12}$ must be between $\sqrt{9}$ and $\sqrt{16.}$

$$\sqrt{9}<\sqrt{12}<\sqrt{16}$$

$$3<\sqrt{12}<4$$

**Step 1:** To find the first estimate, choose the integer that is closest to $\sqrt{12}$.

The integer closest to $\sqrt{12}$ is 3.

The first estimate is 3.

**Step 2:** Divide the radicand by the first estimate.

$$12÷3=4$$

**Step 3:** To find the second estimate, find the average of the quotient in Step 2 and the first estimate.

$$\frac{3+4}{2}=\frac{7}{2}=3.5$$

The second estimate is 3.5.

**Step 4:** Repeat Step 2. But this time, divide the radicand by the second estimate.

$12÷3.5≈3.429$

(Rounded to the nearest thousandths)

**Step 5:** To find the third estimate, repeat Step 3. This time, find the average of the quotient in Step 4 and the second estimate.

$$\frac{3.5+3.429}{2}=\frac{6.929}{2}=3.4645$$

Therefore, the closest approximate of $\sqrt{12}$ is 3.46.

**Example 2:** Approximate $\sqrt{5}$.

Since 5 is between 4 and 9, $\sqrt{5}$ must be between $\sqrt{4}$ and $\sqrt{9.}$

$$\sqrt{4}<\sqrt{5}<\sqrt{9}$$

$$2<\sqrt{12}<3$$

**Step 1:** To find the first estimate, choose the integer that is closest to $\sqrt{5}$.

The integer closest to $\sqrt{5}$ is 2.

The first estimate is 2.

**Step 2:** Divide the radicand by the first estimate.

$$5÷2=2.5$$

**Step 3:** To find the second estimate, find the average of the quotient in Step 2 and the first estimate.

$$\frac{2.5+2}{2}=\frac{4.5}{2}=2.25$$

The second estimate is 2.25.

**Step 4:** Repeat Step 2. But this time, divide the radicand by the second estimate.

$5÷2.25≈2.222$

(Rounded to the nearest thousandths)

**Step 5:** To find the third estimate, repeat Step 3. This time, find the average of the quotient in Step 4 and the second estimate.

$$\frac{2.25+2.222}{2}=\frac{4.472}{2}=2.236$$

Therefore, the closest approximate of $\sqrt{5}$ is 2.236.

**Example 3:** Approximate $\sqrt{24}$.

Since 24 is between 16 and 25, $\sqrt{24}$ must be between $\sqrt{16}$ and $\sqrt{25.}$

$$\sqrt{16}<\sqrt{24}<\sqrt{25}$$

$$4<\sqrt{24}<5$$

**Step 1:** To find the first estimate, choose the integer that is closest to $\sqrt{24}$.

The integer closest to $\sqrt{24}$ is 5.

The first estimate is 5.

**Step 2:** Divide the radicand by the first estimate.

$$24÷5=4.8$$

**Step 3:** To find the second estimate, find the average of the quotient in Step 2 and the first estimate.

$$\frac{4.8+5}{2}=\frac{9.8}{2}=4.9$$

The second estimate is 4.9.

**Step 4:** Repeat Step 2. But this time, divide the radicand by the second estimate.

$24÷4.9≈4.898$

(Rounded to the nearest thousandths)

**Step 5:** To find the third estimate, repeat Step 3. This time, find the average of the quotient in Step 4 and the second estimate.

$$\frac{4.9+4.898}{2}=\frac{9.798}{2}=4.899$$

Therefore, the closest approximate of $\sqrt{24}$ is 4.899.

**Sample Problem 6:** Approximate the square root up to the third estimate by averaging

|  |  |
| --- | --- |
| a) $\sqrt{8}$Since 8 is between 4 and 9, $\sqrt{8}$ must be between $\sqrt{4}$ and $\sqrt{9.}$$$\sqrt{4}<\sqrt{8}<\sqrt{9}$$$$2<\sqrt{8}<3$$Step 1: The integer closest to $\sqrt{8}$ is 3.The first estimate is 3.Step 2: Divide the radicand by the first estimate.$$8÷3≈2.67$$Step 3: To find the second estimate, find the average of the quotient in Step 2 and the first estimate.$$\frac{2.67+3}{2}=\frac{5.67}{2}=4.832$$The second estimate is 4.832.Step 4: Repeat Step 2. But this time, divide the radicand by the second estimate.$$8÷4.832≈0.828$$Step 5: To find the third estimate, repeat Step 3. This time, find the average of the quotient in Step 4 and the second estimate.$$\frac{0.828+4.832}{2}=\frac{5.66}{2}=2.83$$The closest approximate of $\sqrt{8} $is 2.83 | b) $\sqrt{29}$Since 29 is between 25 and 36, $\sqrt{29}$ must be between $25$ and $\sqrt{26.}$$$\sqrt{25}<\sqrt{29}<\sqrt{36}$$$$5<\sqrt{29}<6$$Step 1: The integer closest to $\sqrt{29}$ is 5.The first estimate is 5.Step 2: Divide the radicand by the first estimate.$$29÷5=5.8$$Step 3: To find the second estimate, find the average of the quotient in Step 2 and the first estimate.$$\frac{5.8+5}{2}=\frac{10.8}{2}=5.4$$The second estimate is 5.4.Step 4: Repeat Step 2. But this time, divide the radicand by the second estimate.$$29÷5.4≈5.37$$Step 5: To find the third estimate, repeat Step 3. This time, find the average of the quotient in Step 4 and the second estimate.$$\frac{5.37+5.4}{2}=\frac{10.77}{2}=5.385$$The closest approximate of $\sqrt{29} $is 5.385. |