Name: ____ Period: __________ Date: _______________

Approximating Square Roots

Guide Notes

SQUARE OF A NUMBER

To square a number means to multiply a number by itself.

This small number called "**exponent**" indicates that the base, which is 5, will be a factor twice.

This is read as "5 squared is 25".

25 is obtained by making 5 a factor, twice.

More examples:

0 squared $= 0^2 = 0 \cdot 0 = 0$

1 squared $= 1^2 = 1 \cdot 1 = 1$

-1 squared $= (-1)^2 = -1 \cdot -1 = 1$

2 squared $= 2^2 = 2 \cdot 2 = 4$

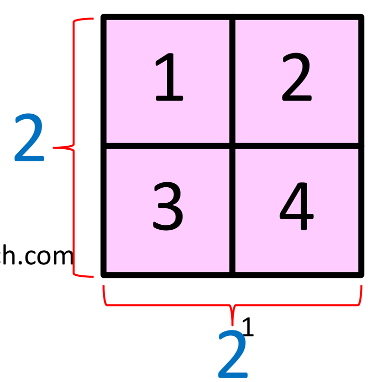
-2 squared $= (-2)^2 = -2 \cdot -2 = 4$

3 squared $= 3^2 = 3 \cdot 3 = 9$

-3 squared $= (-3)^2 = -3 \cdot -3 = 9$

$0, 1, 4, 9, \ldots$ **are called PERFECT SQUARE NUMBERS.**

Relating this to shapes: To create a perfect square, all four sides must have equal measures.



2 squared

Name: ____ Period: __________ Date: _______________

Approximating Square Roots

Guide Notes

Sample Problem 1: Read and answer the questions below:

a. Is 81 a perfect square number? Justify your answer.
Answer:

81 is a perfect square number because it can be obtained by making 9 a factor twice.

b. Is 121 a perfect square number? Justify your answer.
Answer:

121 is a perfect square number because it can be obtained by making 11 a factor twice.

Sample Problem 2: Square the following numbers.

a) 7

$$7^2 = 7 \cdot 7 = 49$$

b) -5

$$(-5)^2 = -5 \cdot -5 = 25$$

c) 8

$$8^2 = 8 \cdot 8 = 64$$

d) -10

$$(-10)^2 = (-10) \cdot (-10) = 100$$

Name: ____ Period: __________ Date: _______________

Approximating Square Roots

Guide Notes

SQUARE ROOT OF A NUMBER

Subtraction is the inverse of addition, and so is division the inverse of multiplication. The inverse of squaring a number is by getting its **SQUARE ROOT**.

The symbol of square root is denoted by the symbol: $\sqrt{\ }$. The symbol is also referred to as the "**radical**" sign. Any number placed inside the radical sign is called a "**radicand**".

If $a^2 = b$, then a number a is a square root of b, $a = \sqrt{b}$.

All positive real numbers have two roots, one positive and one negative. Normally, if we want to indicate that we want to get both roots, we can use the symbol: $\pm\sqrt{\ }$.

Study the examples below:

Since and , then 1 and -1 are both square roots of 1.

Name: ____ Period: ___________ Date: ________________

Approximating Square Roots

Guide Notes

Since and , then 2 and -2 are both square roots of 4.
Since and , then 3 and -3 are both square roots of 9.

$+1\ and\ -1, +2\ and\ -2, +3\ and\ -3$ **can also be written as** $\pm 1, \pm 2\ and\ \pm 3$.

PRINCIPAL SQUARE ROOT

The positive square root of any non-negative real number is called the **principal square root**.

For any non-negative real numbers a and b such that $a^2 = b$, a is called the principal square root of b, denoted by $\sqrt{b}$.

Since $1^2 = 1, then\ \sqrt{1} = 1$

The principal square root of 1 is 1.

Since $2^2 = 4, then\ \sqrt{4} = 2$

Name: ____ Period: __________ Date: _______________

Approximating Square Roots

Guide Notes

The principal square root of 4 is 2.

Since $3^2 = 9, then\ \sqrt{9} = 3$

The principal square root of 9 is 3.

The square root of a negative number is not defined. This means that it has no value at all.

$\sqrt{-1} = undefined$

$\sqrt{-4} = undefined$

$\sqrt{-9} = undefined$

Sample Problem 3: Find the value of the following.

a) $\pm\sqrt{49} = \pm 7$

b) $\sqrt{-49}$ undefined

c) $\sqrt{121} = 11$

d) $\pm\sqrt{16} = \pm 4$

e) $\sqrt{100} = 10$

f) $\pm\sqrt{64} = \pm 8$

Name: ____ Period: __________ Date: _______________

Approximating Square Roots

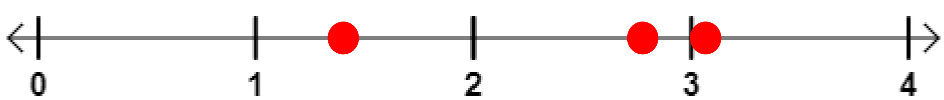
Guide Notes

APPROXIMATING THE SQUARE ROOT OF A NUMBER

The square root of perfect square numbers like 1, 4, 9, 16, 25, and 36 are rational numbers. Non-perfect square numbers on the other hand have irrational numbers as roots. By irrational, it means that the square roots cannot be expressed as a ratio of two integers.

5 can be expressed as a ratio of two integers:	It can't be expressed as a ratio of two integers:

Irrational numbers complete the set of real numbers. They fill up the spaces between rational numbers. This can be visualized using a number line.



Look at the approximate square roots of some non-perfect square numbers.

$$\sqrt{2} = 1.414\ldots$$

Name: ____ Period: __________ Date: ______________

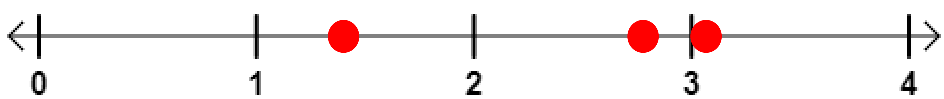
Approximating Square Roots

Guide Notes

$\sqrt{8} = 2.828 \ldots$

$\sqrt{10} = 3.162 \ldots$

The square root of a non-perfect square number can be found between two rational numbers. It can be approximated by looking for two consecutive integers or rational numbers where the square root lies in between.



$\sqrt{2}$ is found between rational numbers 1and 2.

$\sqrt{8}$ is found between rational numbers 2 and 3.

$\sqrt{10}$ is found between rational numbers 3 and 4.

Example 1: Find two consecutive integers between which $\sqrt{12}$ lies.

Step 1: Examine the radicand.

The radicand is 12.

Step 2: Look for a perfect square number, less than and closest to the radicand.

Name: ____ Period: __________ Date: _______________

Approximating Square Roots

Guide Notes

The closest perfect square number less than 12 is 9.

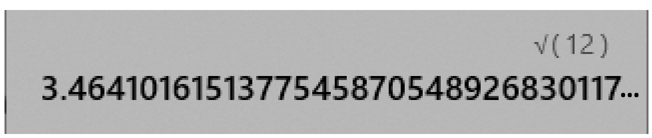
Step 3: Look for a perfect square number, greater than and closest to the radicand.

The closest perfect square number greater than 12 is 16.

$$\sqrt{9} < \sqrt{12} < \sqrt{16}$$
$$3 < \sqrt{12} < 4$$

Therefore, the value of $\sqrt{12}$ is between rational numbers 3 and 4.

Grab a calculator and confirm to confirm the answer:



$$\sqrt{12} \approx 3.464 \ldots$$

$3.464 \ldots$ **is between 3 and 4.**

Example 2: Find two rational numbers with two decimal places between which $\sqrt{7}$ lies.

Step 1: Examine the radicand.

The radicand is 7.

Step 2: Look for a perfect square number, less than and closest to the radicand.

Name: ____ Period: __________ Date: _______________

Approximating Square Roots

Guide Notes

The closest perfect square number less than 7 is 4.

Step 3: Look for a perfect square number, greater than and closest to the radicand.

The closest perfect square number greater than 7 is 9.

$$\sqrt{4} < \sqrt{7} < \sqrt{9}$$
$$2 < \sqrt{7} < 3$$

Therefore, the value of $\sqrt{7}$ is between rational numbers 2 and 3.

To find two rational numbers with two decimal places between which $\sqrt{7}$ lies, let's do some estimation. Since $\sqrt{7}$ lies between 2 and 3, we can square the decimal numbers between 2 and 3 to start the estimation.

$$2.1^2 = 4.41$$
$$2.2^2 = 4.84$$
$$2.3^2 = 5.29$$
$$2.4^2 = 5.76$$
$$2.5^2 = 6.25$$
$$2.6^2 = 6.76$$
$$2.7^2 = 7.29$$

So, lies between 2.6 and 2.7.

To find the closest approximate square root, let's continue the estimation. Since we are looking for two rational numbers with two decimal places between which $\sqrt{7}$ lies, we'll use the decimal numbers between 2.6 and 2.7.

$$2.61^2 = 6.8121$$
$$2.62^2 = 6.8644$$

So, lies between 2.64 and 2.65.

Name: ____ Period: __________ Date: _______________

Approximating Square Roots

Guide Notes

$$2.63^2 = 6.9619$$
$$2.64^2 = 6.9696$$
$$2.65^2 = 7.0225$$

So, the value of $\sqrt{7}$ is between rational numbers with two decimal places 2.64 and 2.65.

Example 3: Find two rational numbers with two decimal places between which $\sqrt{18}$ lies.

*(*Note: follow the same steps as we did in the previous examples.)*

Since 18 is between 16 and 25, $\sqrt{18}$ must be between $\sqrt{16}$ and $\sqrt{25}$.

$$\sqrt{16} < \sqrt{18} < \sqrt{25}$$
$$4 < \sqrt{18} < 5$$

By estimation, we have:

$$4.1^2 = 16.81$$
$$4.2^2 = 17.64$$
$$4.3^2 = 18.49$$

So, lies between 4.2 and 4.3.

To find the two rational numbers with two decimal places between which $\sqrt{18}$ lies, let's estimate further:

$$4.21^2 = 17.7241$$
$$4.22^2 = 17.8084$$
$$4.23^2 = 17.8929$$
$$4.24^2 = 17.9776$$
$$4.25^2 = 18.0625$$

So, lies between 4.24 and 4.25.

Name: ____ Period: ___________ Date: ________________

Approximating Square Roots

Guide Notes

So, the value of $\sqrt{18}$ is between rational numbers with two decimal places 4.24 and 4.25.

Sample Problem 4: Find two consecutive integers between which the given square root lies.

a) $\sqrt{11}$

The radicand is 11.

The closest perfect square number less than 11 is 9.

The closest perfect square number greater than 11 is 16.

$$\sqrt{9} < \sqrt{11} < \sqrt{16}$$
$$3 < \sqrt{11} < 4$$

The square root of $\sqrt{11}$ is between 3 and 4.

b) $\sqrt{35}$

The radicand is 35.

The closest perfect square number less than 35 is 25.

The closest perfect square number greater than 35 is 36.

$$\sqrt{25} < \sqrt{35} < \sqrt{36}$$
$$5 < \sqrt{35} < 6$$

The square root of $\sqrt{35}$ is between 5 and 6.

c) $\sqrt{28}$

d) $\sqrt{91}$

Name: ______________________________ Period: __________ Date: ______________

Approximating Square Roots

Guide Notes

The radicand is 28.

The closest perfect square number less than 28 is 25.

The closest perfect square number greater than 28 is 16.

$$\sqrt{25} < \sqrt{28} < \sqrt{36}$$
$$5 < \sqrt{28} < 6$$

The square root of $\sqrt{28}$ is between 5 and 6.

The radicand is 91.

The closest perfect square number less than 91 is 81.

The closest perfect square number greater than 91 is100.

$$\sqrt{81} < \sqrt{91} < \sqrt{100}$$
$$9 < \sqrt{91} < 10$$

The square root of $\sqrt{91}$ is between 9 and 10.

Sample Problem 5: Determine two rational numbers with two decimal places between which the given square root lies.

a) $\sqrt{5}$

Since 5 is between 4 and 9, $\sqrt{5}$ must be between $\sqrt{4}$ and $\sqrt{9}$.

$$\sqrt{4} < \sqrt{5} < \sqrt{9}$$
$$2 < \sqrt{5} < 3$$

By estimation, we have:

$$2.1^2 = 4.41$$
$$2.2^2 = 4.48$$
$$2.3^2 = 5.29$$

So, lies between 2.2 and 2.3.

b) $\sqrt{23}$

Since 23 is between 16 and 25, $\sqrt{23}$ must be between $\sqrt{16}$ and $\sqrt{25}$.

$$\sqrt{16} < \sqrt{23} < \sqrt{25}$$
$$4 < \sqrt{23} < 5$$

By estimation, we have:

$$4.6^2 = 21.16$$
$$4.7^2 = 22.09$$
$$4.8^2 = 23.04$$

So, lies between 4.7 and 4.8.

Name: ______________________________ Period: __________ Date: ______________

Approximating Square Roots

Guide Notes

To find the two rational numbers with two decimal places between which $\sqrt{5}$ lies, let's estimate further:

$2.21^2 = 4.8841$
$2.22^2 = 4.9284$
$2.23^2 = 4.9729$
$2.24^2 = 5.0176$

So, lies between 2.23 and 2.24.

The square root of $\sqrt{5}$ is between 2.23 and 2.24.

To find the two rational numbers with two decimal places between which $\sqrt{23}$ lies, let's estimate further:

$4.76^2 = 22.6576$
$4.77^2 = 22.7529$
$4.78^2 = 22.8484$
$4.79^2 = 22.9441$
$4.80^2 = 23.04$

So, lies between 4.79 and 4.80.

The square root of $\sqrt{23}$ is between 4.79 and 4.80.

APPROXIMATING THE SQUARE ROOT OF A NUMBER BY AVERAGING

Another strategy to approximate square roots is by averaging. This is a less complicated method that involves square root estimates. Study the examples below:

Example 1: Approximate $\sqrt{12}$.

Since 12 is between 9 and 16, $\sqrt{12}$ must be between $\sqrt{9}$ and $\sqrt{16}$.

$$\sqrt{9} < \sqrt{12} < \sqrt{16}$$
$$3 < \sqrt{12} < 4$$

Name: ____ Period: __________ Date: _______________

Approximating Square Roots

Guide Notes

Step 1: To find the first estimate, choose the integer that is closest to $\sqrt{12}$.

The integer closest to $\sqrt{12}$ is 3.

The first estimate is 3.

Step 2: Divide the radicand by the first estimate.

$$12 \div 3 = 4$$

Step 3: To find the second estimate, find the average of the quotient in Step 2 and the first estimate.

$$\frac{3 + 4}{2} = \frac{7}{2} = 3.5$$

The second estimate is 3.5.

Step 4: Repeat Step 2. But this time, divide the radicand by the second estimate.

$$12 \div 3.5 \approx 3.429$$

(Rounded to the nearest thousandths)

Step 5: To find the third estimate, repeat Step 3. This time, find the average of the quotient in Step 4 and the second estimate.

$$\frac{3.5 + 3.429}{2} = \frac{6.929}{2} = 3.4645$$

Therefore, the closest approximate of $\sqrt{12}$ is 3.46.

Example 2: Approximate $\sqrt{5}$.

Name: ____ Period: __________ Date: _______________

Approximating Square Roots

Guide Notes

Since 5 is between 4 and 9, $\sqrt{5}$ must be between $\sqrt{4}$ and $\sqrt{9}$.

$$\sqrt{4} < \sqrt{5} < \sqrt{9}$$
$$2 < \sqrt{12} < 3$$

Step 1: To find the first estimate, choose the integer that is closest to $\sqrt{5}$.

The integer closest to $\sqrt{5}$ is 2.

The first estimate is 2.

Step 2: Divide the radicand by the first estimate.

$$5 \div 2 = 2.5$$

Step 3: To find the second estimate, find the average of the quotient in Step 2 and the first estimate.

$$\frac{2.5 + 2}{2} = \frac{4.5}{2} = 2.25$$

The second estimate is 2.25.

Step 4: Repeat Step 2. But this time, divide the radicand by the second estimate.

$$5 \div 2.25 \approx 2.222$$

(Rounded to the nearest thousandths)

Step 5: To find the third estimate, repeat Step 3. This time, find the average of the quotient in Step 4 and the second estimate.

Name: ____ Period: __________ Date: _______________

Approximating Square Roots

Guide Notes

$$\frac{2.25 + 2.222}{2} = \frac{4.472}{2} = 2.236$$

Therefore, the closest approximate of $\sqrt{5}$ is 2.236.

Example 3: Approximate $\sqrt{24}$.

Since 24 is between 16 and 25, $\sqrt{24}$ must be between $\sqrt{16}$ and $\sqrt{25}$.

$$\sqrt{16} < \sqrt{24} < \sqrt{25}$$

$$4 < \sqrt{24} < 5$$

Step 1: To find the first estimate, choose the integer that is closest to $\sqrt{24}$.

The integer closest to $\sqrt{24}$ is 5.

The first estimate is 5.

Step 2: Divide the radicand by the first estimate.

$$24 \div 5 = 4.8$$

Step 3: To find the second estimate, find the average of the quotient in Step 2 and the first estimate.

$$\frac{4.8 + 5}{2} = \frac{9.8}{2} = 4.9$$

Name: ____ Period: __________ Date: _______________

Approximating Square Roots

Guide Notes

The second estimate is 4.9.

Step 4: Repeat Step 2. But this time, divide the radicand by the second estimate.

$$24 \div 4.9 \approx 4.898$$

(Rounded to the nearest thousandths)

Step 5: To find the third estimate, repeat Step 3. This time, find the average of the quotient in Step 4 and the second estimate.

$$\frac{4.9 + 4.898}{2} = \frac{9.798}{2} = 4.899$$

Therefore, the closest approximate of $\sqrt{24}$ is 4.899.

Name: ____ Period: __________ Date: ______________

Approximating Square Roots

Guide Notes

Sample Problem 6: Approximate the square root up to the third estimate by averaging

a) $\sqrt{8}$

Since 8 is between 4 and 9, $\sqrt{8}$ must be between $\sqrt{4}$ and $\sqrt{9}$.

$$\sqrt{4} < \sqrt{8} < \sqrt{9}$$
$$2 < \sqrt{8} < 3$$

Step 1: The integer closest to $\sqrt{8}$ is 3.
The first estimate is 3.

Step 2: Divide the radicand by the first estimate.

$$8 \div 3 \approx 2.67$$

Step 3: To find the second estimate, find the average of the quotient in Step 2 and the first estimate.

$$\frac{2.67 + 3}{2} = \frac{5.67}{2} = 4.832$$

The second estimate is 4.832.

Step 4: Repeat Step 2. But this time, divide the radicand by the second estimate.

$$8 \div 4.832 \approx 0.828$$

Step 5: To find the third estimate, repeat Step 3. This time, find the average of the quotient in Step 4 and the second estimate.

$$\frac{0.828 + 4.832}{2} = \frac{5.66}{2} = 2.83$$

b) $\sqrt{29}$

Since 29 is between 25 and 36, $\sqrt{29}$ must be between 25 and $\sqrt{26}$.

$$\sqrt{25} < \sqrt{29} < \sqrt{36}$$
$$5 < \sqrt{29} < 6$$

Step 1: The integer closest to $\sqrt{29}$ is 5.
The first estimate is 5.

Step 2: Divide the radicand by the first estimate.

$$29 \div 5 = 5.8$$

Step 3: To find the second estimate, find the average of the quotient in Step 2 and the first estimate.

$$\frac{5.8 + 5}{2} = \frac{10.8}{2} = 5.4$$

The second estimate is 5.4.

Step 4: Repeat Step 2. But this time, divide the radicand by the second estimate.

$$29 \div 5.4 \approx 5.37$$

Step 5: To find the third estimate, repeat Step 3. This time, find the average of the quotient in Step 4 and the second estimate.

$$\frac{5.37 + 5.4}{2} = \frac{10.77}{2} = 5.385$$

Name: ____ Period: ___________ Date: _________________

Approximating Square Roots

Guide Notes

The closest approximate of $\sqrt{8}$ is 2.83

The closest approximate of $\sqrt{29}$ is 5.385.